Water Resources Br.

TORONTO HARBOUR NUMERICAL MODEL

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INTRODUCTION

As part of the overall Toronto Harbour study, a two-dimensional numerical model of currents and total dissolved solids (as measured by conductivity) in the inner harbour has been developed. Such a model has its greatest value in its predictive capacity. The effect of water management decisions such as intake and outfall location, landfilling and changes in harbour configurations and gaps can be simulated with a model of this nature. This is important as these projects involve large capital expenditures and considerable potential for water quality changes.

Concentration contouring of physical-chemical parameters such as conductivity and dissolved oxygen can be developed for various discharge conditions, outflows, dispersion, meteorological conditions and changes in harbour geometry.

For such predictions to be realistic, however, the model must be capable of considering phenomena which occur at very short time intervals (order of minutes). That such phenomena are important has been demonstrated in the case of Hamilton Harbour (MOE, 1974; Poulton and Palmer, 1976), where changes in conductivity of several hundred umho, and in dissolved oxygen of over 50% saturation, have been shown to occur within minutes. In the case of Toronto Harbour, Simpson (in Institute of Environmental Studies, 1974) states that short periodicities have little effect in enhancing water quality. With this consideration in mind, recording chemistry and current meters have been operated for extended periods in the harbour and the real-time data thus obtained have been used directly in the numerical model. Internal points have also been instrumented and the results used for calibration and verification

purposes. The present report describes the calibration of the model in terms of the physical constants used and the size of the time step required for stable solutions. Computerized contouring methods have also been applied for pictorial handling of spatial variations.

DESCRIPTION OF MODEL

With the exception of real-time input for the principal source and boundaries, the numerical model used is identical to that used in the Hamilton Harbour Study (MOE, 1974). The model is based on a two-dimensional estuarine water quality model initially developed and verified by J. Leendertse (1970, 1971) for Jamaica Bay, New York City. The depth-integrated equations for momentum, continuity and mass balance are coupled and solved by finite-difference techniques in a space-staggered grid. Although a three-dimensional numerical model of Toronto Harbour is presently under development, the initial work on Toronto Harbour was conducted with a two-dimensional model. Such a model was felt to be adequate to describe the general circulation and water quality patterns in the inner harbour in the absence of thermal stratification (late fall); indeed, the presently available input data set is insufficient to calibrate and verify a three-dimensional model. However, the application of real-time data inputs to this model constitutes a real "first" as previous model studies generally used only mean daily loadings and hourly lake exchange values. Furthermore, the calibration studies involving values of the physical constants reported here will prove useful when the three-dimensional model currently under development becomes operational.

The hydrodynamic (currents and water levels) portion of the model takes into account advection of momentum, water level variation, Coriolis force, wind stress, bottom friction and the effects on momentum and volume of the addition or subtraction of water at intakes and outfalls.

The hydrostatic approximation is used and dispersion of momentum is ignored. The mass transport portion of the model takes into account advection, dispersion, and change of mass at intakes or outfalls. The equations and method of solution have been outlined (MOE, 1974) and will not be repeated here.

Input Data

A computational grid of space step length 152 m (500 ft) was prepared from a Canadian Hydrographic Service chart of Toronto Harbour, analogous to the Hamilton Harbour model. The grid used is shown in Figure 1. The east and west gaps were treated as open boundaries, while the Keating Channel (Don River) was considered a source and the ship canal (Hearn generating station intake) a sink. To supply real-time data for input to the model, the two gaps were monitored with digitally recording water chemistry and current meters; currents were recorded every 10 min, and conductivity every 20 min. The Keating Channel was monitored with a chemistry meter; as shipping considerations precluded mooring a current meter in the channel, streamflow data from the Don River gauge at Todmorden recorded at a 60 minute interval were used as input to the model. To check the validity of using flow data from one location and chemistry from another, the time series from the two locations were cross-correlated. Cross-correlation between streamflow and conductivity tended to indicate only a very weak relationship between the gauge and chemistry monitor. For example, two periods of increased flow (Figure 3) showed opposite correlations with conductivity changes, likely related to runoff-caused dilution. Consequently, no time adjustment was used with these data. Locations of meter installtions are given in Figure 2; complete details of meter installations and operation are given in a separate report on meter operations (MOE, 1977).

No data were available for the Hearn ship channel on a real-time basis; indeed, average figures available varied over a tremendous range. Available flow values can be summarized as follows:

Source	Flow (m^3s^{-1})
Simpson (Subcommittee Report)	17-42; average 29
Hearn generating station, November 1975 (H. Makuch, personal comm.)	144
Lake Systems Unit (July 1976, unpublished)	75-100

With this large range of figures available, it was decided that an acceptable figure would be one that allowed maintenance of a near zero change of average water level during the modelled period. Tests indicated such a figure to be 50 m^3s^{-1}.

This figure may be smaller than the true figure as storm water overflows were neglected in the first runs; the Toronto Central Waterfront Water Environment Study indicated several storm sewer overflows to the inner harbour with a total estimated flow of approximately 35 m^3/s^{-1} (Subcommittee Report, p. 13). Although these flows were not considered, the major portion of the storm water flow occurs in the Don River, which is considered in the model. On the other hand, as these flows represent a significant volume, the major flows should be monitored carefully in future surveys and included in the modelling efforts.

Hourly values of wind data measured at the Toronto Island Airport were used as input to the hydrodynamic model. The wind stress coefficient relating wind speeds to surface stress was given the same value (0.0032) used in the Hamilton Harbour study (MOE, 1974).

A modelling period was selected in which the three pairs of recording current and chemistry meters were all functioning well. The record also included two rainstorm events which produced significant loadings from the Don River. The selected

period was November 7-11, 1975, beginning at 1300 hours on November 7. The flow-conductivity relationship occurring in this period is shown in Figure 3. Hourly values for November 7-8 used in the model are summarized in Table 1; current and chemistry meter data at 10-20 minute intervals though used in the model are not included for the sake of brevity. In addition, the average wind values used are summarized in Table 2.

Selection of Model Parameters

Initial attempts at operating the model in Toronto Harbour utilized model physical parameters taken from the Hamilton Harbour model study (MOE, 1974). However, as this was the first effort at operating a model with real-time data input at intervals of the order of minutes, and since large variations in currents and water chemistry are known to occur within minutes, it was felt tht a thorough investigation of model parameters was necessary for proper operation of the numerical model. This involved the checking of the hydrodynamic time step length, effect of wind stress on a moving surface, velocity techniques at impervious boundaries, depth and bottom roughness interrelations and the concentration time step length.

Although the time step lengths of 2 or 2.5 min. as used in the initial Hamilton Harbour model runs produced numerically stable results, Williams and Hinwood (1976) have indicated the need for a shorter time step to achieve replication of modelled results. Following work of Leendertse, they suggest the following criterion for the time step length t:

$$1 \leq \frac{\Delta t}{\Delta x} \sqrt{gh} \leq 2.5$$

where

Δx = space step length

g = acceleration due to gravity

h = average or maximum water depth.

Their own work suggested a value closer to 1, rather than 2.5, was required. For Toronto Harbour, for a water depth of 10 m, this criterion indicates that a time step length of the order of 15 to 37.5 s should be used. The model results should obviously not be a function of the time step length within this range. However, decreasing the time step length below 30 s produced rapidly increasing instabilities in which currents of over 100 cm s^{-1} in either direction could occur within minutes of each other, clearly a physically unrealistic situation. In order to resolve this problem, the various forcing functions of the model were examined in detail.

As wind is the major forcing function for water movements, it was examined first. The wind vector should be a difference vector which considers surface water currents (J.B. Hinwood, personal comm.). Model results after 18 hr for time steps of 30, 15 and 7.5 s respectively (Figure 4) show some decrease in observed velocities but physically unrealistic values of up to 72 cm s^{-1} were still obtained.

Examining the magnitude of the terms in the momentum equation at different modelled times revealed large variations in the advection terms near closed boundaries. This suggested alteration of the finite-difference approximations of these terms to replace velocities for points which represented land points, with adjacent water points. However, this procedure yielded no improvement in model results.

On the other hand, success was achieved by increasing the water depth in shallow zones. As the wind stress term is inversely proportional to the water depth H, relatively high values of this term were found at points of very shallow depth, creating high velocities at these points which were gradually propagated to adjacent points. The model was therefore run with a minimum water depth of 4 m and time step lengths of 30, 15 and 7.5 s.

A sample output of v velocities (velocity component parallel to Western Gap) as a function of time after 18 hr is given in Figure 5, the dotted lines again indicating zero velocity. Obviously, 30 seconds is too long but 15 seconds was acceptable as it replicates the result at the shortest time step length. Next a time step length of 20 seconds was tested; the result (Figure 6) showed that this was also acceptable. Therefore, 20 seconds was chosen as the longest acceptable time step length in Toronto Harbour.

However, the problem remained that the input data represented a considerably deepened version of the harbour next to the islands along the southern and southwestern shores. In order to improve this situation, the minimum water depth was reduced to 2 m and the effect of bottom friction was next examined.

Bottom friction is ordinarily determined in the field by comparing a measured velocity profile to the wind stress results. Although some velocity profiles were measured in Hamilton Harbour during 1976, the data were too variable to permit any meaningful calculation of Chezy's C or Manning's n.

A literature survey of typical bottom friction equations was conducted; all figures were converted to Mannings n values in $m^{-1/3}s$. Typical results are as follows:

Location	Manning's n	Reference
Westernport Bay, Australia	0.052	Williams & Hinwood (1976)
Lake Ontario	0.05	Simons (1971)
Jamaica Bay, N.Y.	0.034-0.038	Leendertse and Gritton (1971)
San Francisco Bay	0.130	Leendertse and Liu (1975)
Canal lined with concrete	0.018	Chow (1959)
Irrigation canal, sand bottom	0.030	Ibid
Cobble-bottom channel	0.042	Ibid
River channel with even bottom	0.052	Ibid
Channel with irregular sides and bottom, with growth of weeds and grass	0.074	Ibid
Irregular channel with dense growths of willows and weeds	0.119	Ibid

Without detailed field measurements, Manning's n must be estimated by the fit of modelled data to observed data. Considering the range of values given above, and the bottom topography of Toronto Harbour, a grid consisting of four n values ranging from 0.040 (in the dredged part of the harbour) to 0.060 (in the shallow southeastern part of the harbour) was established. This grid is shown in Figure 7. Although these values are arbitrary, they do reflect the known fact that shallow waters tend to have higher n values and provide at least an estimate for a parameter which is difficult to measure except in river systems. With this grid the maximum velocity after 24 hours was 12 cm s^{-1}. This is close to the maximum measured velocity of 11.3 cm s^{-1}.

Effect of concentration time step length

A 24 hour model run on Toronto Harbour with a time step length of 20 s, requires 13 minutes of CPU time with 120K core on an IBM S/370-165 computer. The high cost of this computer time renders it difficult to model for an extended period of time, alter physical parameters (bottom roughness, water depth, time step length, etc.) for model efficiency, or to test the various alternatives (harbour geometry, loading, etc.) that the model is designed for. Anticipated costs for Hamilton Harbour will be much higher due to the larger modelled area, unless the space step is increased to an unreasonably high value.

The principal mechanism of mass distribution in the mass balance equations is advection. At the maximum velocities (15-25 cm s^{-1}) generally observed from 12 months of current meter data and a grid spacing of 152 m, one expects that a longer time step could be utilized in the model to save computer time. To this end, mass balance time step lengths of 3, 5 and 10 times the selected hydrodynamic step length of 20s were tried. Some sample results are given in Table 3 as time series of concentrations observed over a 2 hour interval at three points in the harbour. The mass balance time step of 200 s was unacceptable as gradual cyclic changes in concentration were produced. The step lengths of 60 and 100 s both produced small random variations which were almost always below 0.5 mg/l except at the shallowest areas (Table 3b), where deviations of 1.0-1.5 mg/l sometimes occurred, especially at 100 s.

It should be emphasized that the only justification in operating a mass balance time step longer than the hydrodynamic time step length is economics of computer time. The time step length chosen will represent a balance between accuracy and economics. Although operating at a 60 s mass balance time step

length produces larger variations (± 0.5 mg/l) which are larger than those produced when the hydrodynamic and mass balance time steps are equal (± 0.1 mg/l), these variations are smaller than either measurement precision (1 umho or 0.65 mg/l) or observed daily variations in the central harbour (daily standard deviations of conductivity in November 1975 were 2 to 5 umho, or a TDS of 1.3 to 3 mg/l). If these observed variations are objectionable during cross correlation with field data, time series smoothing using binomial weights could be applied.

The concept of increased mass balance time step has been successfully used by Wickramaratne et al (1976). They used a stability criterion based on advection as follows:

$$\max \left[|u| + |v| \right] \frac{\Delta t}{\Delta x} \leq 1$$

Although they were able to use a mass balance time step of 15 times their hydrodynamic time step, they were dealing with ocean currents which are primarily tidally driven. In Toronto Harbour, preliminary hydrodynamic model results contained a principal periodicity of 8 to 10 minutes likely due to harbour oscillation. Cumulative effects from this oscillation are probably responsible for the deviations observed as the water quality time step length is increased.

MODEL RESULTS

With the factors affecting model performance as discussed above, a numerical model run for 72 hours was performed, using input data of November 7-10, 1975. Pertinent input parameters used are summarized as follows:

1. Average wind field according to Table 2.
2. Wind stress coefficient θ = 0.0032.
3. Manning's n values according to Figure 7.
4. Hydrodynamic time step = 20 sec.
5. Mass balance time step = 60 sec.
6. Minimum harbour water depth = 2 m.
7. Mass balance dispersion coefficient = 0.93 $m^2 sec^{-1}$

As preliminary operations (Figures 5 to 6) indicated that significant periodicities of the order of minutes exist, velocity (u and v), water level and concentration results were saved on tape every 2 min during the model run.

TIME SERIES ANALYSIS OF MODEL PERIODICITIES

A set of model u and v velocity data at a 2 min interval from row 16, column 8 of the model grid (near current meter location 129, Figure 2) was analyzed by standard time series techniques (Jenkins and Watts, 1968). The results are shown in Figures 8 and 9. Most of the variance is concentrated in peaks of 12.8, 8.4, 6.3, 4.7 and 4.3 min, which were generally significant at the 99% confidence level. Although some of the observed periods may represent aliasing of additional periodicities at periods of less than 4 min, these periodicities were in rough agreement with calculated harbour oscillation periods T_n using the Merian equation for a symmetrical concave parabolic basin (Hutchinson, 1957, p. 304):

$$T_n = \frac{\pi \ell}{(n[n+1] g \; Z_m)^{\frac{1}{2}}}$$

where:

n = order of oscillation

g = acceleration due to gravity

z_m = maximum depth (10 m was used in calculation)

ℓ = length (3200 m for longitudinal and 1700 m for transverse oscillations)

Observed and predicted periodicities are given in Table 4. Model periods are separated on the basis that transverse oscillations have a stronger u velocity component and longitudinal oscillations a stronger v velocity component. Differences between observed and predicted periodicities reflect the complexity of the actual basin.

MODEL PREDICTIONS OF CURRENTS AND WATER QUALITY

Due to the presence of these oscillations, half-hourly average values of currents and water quality were computed to give an overall trend. Predicted currents are shown in Figure 10A for conditions of predominantly west winds, and Figure 10B for conditions of predominantly east winds. The overall current picture in both cases shows the strongest currents following the wind direction along the shallow south shore, with a weaker return current occurring along the north shore. An overall counter-clockwise circulation is present under westerly winds, while an overall clockwise circulation occurs with easterly winds.

A computerized plotting representation of water quality contours predicted by the model is presented in Figures 11a-11i for 6-hour intervals beginning from 24 hours of model time. An initial slow spread of the 225 mg/l total dissolved solids (TDS) contour increases during the first portion of the period of influence of north-east to easterly winds (36 to 48 hours) with most of the harbour having TDS above 225 mg l^{-1} at 48 to 54 hours. Subsequently, the advective effect of the return flow along the north shore combined with a strong inflow of lake water through the Western Gap causes a considerable retreat of the 225 mg/l area to cover only a small northeastern part of the harbour at 72 hours. The area of strong influence from the Don River (defined by the 250 mg/l contour) remains in the northeastern corner of the harbour at all times. The influence of lake water entering the water at the Western Gap is shown by the 216 mg/l contour, which reaches a maximum extent at 30 hours and retreats as the water with higher TDS advances across the harbour. This expansion and contraction of concentration contours is very significant as it demonstrates the non-steady condition of the harbour as shown by the model. The spatial variation of total dissolved solids is in qualitative agreement with the average of a series of 1973

conductivity measurements by the Great Lakes Division, Institute for Environmental Studies, University of Toronto (Figure 12). These results show the lake water effect (less than 360 umho) in the northwest corner, the Don River effect (greater than 370 umho) in the northeast part of the harbour and relatively uniform values (363-370 umho) throughout the central and southern areas.

VERIFICATION OF MODEL OUTPUT: CROSS CORRELATION

In principle, model verification involves a large number of computer runs, each based on a different combination of input parameters such as surface and bottom stress, dispersion, etc. In order to present the capabilities of the present model, the variable parameters were set as already described and a run of 72 hr duration performed. Cross-correlation procedures similar to those used between current and water quality meter installations at the same point, or meter locations at nearby points (Poulton and Palmer, 1976), were adapted to a test of model validity.

Three sets of cross-correlation were performed between the model and the meter data; these were: model u velocities with current meter north speeds; model v velocities with current meter east speeds; and model concentrations with conductivity. Current meter directions were rotated so that north was parallel to the Y axis of the model grid (i.e. 55° east of true north), and the left - handed model co-ordinate system was introduced into the cross correlation program so that all proper correlations would be positive.

In addition to the model parameters already mentioned, four parameters can be varied which are pertinent to the cross-correlation processes. These are:

(a) Location in model grid from which model data were taken.

(b) Number of lags.

(c) Number of binomial smoothing weights on model data.

(d) Start time of meter data.

With regard to model grid locations, eight locations were selected for cross-correlation. These are shown in Figure 13. Locations A to F were selected for cross-correlation with current meter data obtained at location 129; locations G and H were selected for cross-correlation with chemistry meter data at location 128. Preliminary attempts at current meter cross-correlation indicated a tendency toward some negative correlation; as spatial plots of modelled currents tended to support eastward flows close to the south shore and the eastward components predominated in the location 129 current meter data, locations D, E and F were selected for current meter cross-correlation. In addition, some averages of adjacent points were used.

Most authors (for example Shastry, Fan and Erickson, 1972) recommend that the minimum number of lags be 10% of the series length; Fee (1969) suggests 10-15%. Most examples quoted by Jenkins and Watts (1968) used a number of lags equal to about 8 to 10% of the series length. As the field time series corresponding to 72 hours of modelled data are quite short (chemistry, 144 data points; currents, 216 data points) it was decided to start slightly above 10% of the number of data points and move downward if needed. For currents, 24 lags were selected and for chemistry, 20 lags were used. If the maximum cross-correlation occurs at a lag near zero, the coherence is recomputed with the cross-correlation function realigned to a lag of zero, as was done by Lee (1972) and Jenkins and Watts (1968).

To reduce the effect of aliasing of short-term periodicities in the model data (Figure 8 and 9), the model currents (at a 2 min

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To reduce the effect of aliasing of short-term periodicities in the model data (Figure 8 and 9), the model currents (at a 2 min

time interval) were smoothed with nine binomial weights and the chemistry with seven binomial weights prior to sub-sampling at the 20 minute (currents) and 30 minute (chemistry) frequencies of field data. Variation of this parameter within the range above did not produce a large effect on the observed correlations.

In some cases, maximum cross-correlations were initially obtained at lags other than zero. This is possible since the model interpolates wind data which are already hourly-integrated values. In addition, the exact timing of current meter records (both input and verifiction) is often uncertain. These digital records are sequenced by a fixed-interval timer and later converted to real time; hence, the times are approximate. Evidence for the latter effect exists in the variable lags of the maximum cross-correlation results observed in the western gap during the model period (see report on Water Chemistry Meters in Toronto Harbour; this effect was allowed for in setting up input data to the model).

The best results achieved in cross-correlating model data with field data are summarized in Table 5. Plots of coherence spectra for u and v velocities, and concentrations appear in Figures 14-16, respectively. In each case, significant coherences were obtained for a portion of the accessible periods (greater than 1.5 hours for velocities and 2.0 hours for chemistry).

Ideally, significant coherences should be obtained for the entire frequency domain for a perfect simulation. The cross-correlation method is a very hard test for the model and the results indicate the importance of defining the model input constants such as bottom roughness and surface drag, which are both temporal and spatial variables. In addition, the results emphasize the need for metered data (both input and verification) of high quality. Observed data at location 129

indicated zero speeds about 30% of the time and sudden changes from over 10 cm s^{-1} to zero. In addition, many of the compass readings in the eastern gap indicated a flow perpendicular to channel which is physically unrealistic. To maintain constant water level in the harbour during modelling it was necessary to assume that this condition represented an outflow of water from the harbour. This has undoubtedly affected the performance of the model with the present data. However, it must be remembered that the maintenance of 4 or 5 pairs of digitally recording current and water quality meters in simultaneous accurate operation is in itself a difficult task, one that must be accomplished properly in order to operate models of this type.

If the model is working properly, the spectral analysis of model data and measured data carried out separately should show similar periodicities. Even with smoothing as carried out in this analysis, the model spectra are heavily contaminated with aliasing of high frequency peaks (see Figures 8 and 9) when sampled at 20-30 minute intervals. For realistic comparison of model data with field data, a continous recording current meter is desirable. Use of a hot film anemometer to derive high-frequency kinetic energy spectra, as has been done in lake Ontario (Palmer, 1973) is desirable. Given data of this nature, adjustment of model physical parameters and verification of the model should become less of an onerous task.

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TABLE 1: Model input data, November 7-8, 1975.

Date	Time	East Gap Current cm/sec	East Gap Cond. umho/cm	West Gap Current cm/sec	West Gap Cond. umho/cm	Keating Channel Flow m^3/sec	Keating Channel Cond. umho/cm	Wind Speed m/sec	Wind Direction (from)
Nov 7	13.00	1.40	333	1.37	320	1.59	900	3.1	ESE
	14.00	0.68	333	2.47	320	1.59	900	4.0	S
	15.00	0.67	334	-0.37	320	1.61	900	3.1	ESE
	16.00	4.03	333	-3.97	320	1.68	900	2.7	SSE
	17.00	2.90	335	4.26	319	1.75	828	2.7	E
	18.00	1.11	334	7.90	319	1.86	746	0.4	SW
	19.00	1.09	337	6.63	320	1.99	718	5.4	SSW
	20.00	0.63	336	6.07	320	2.30	616	6.7	SSW
	21.00	1.38	335	8.20	320	3.71	814	3.6	WSW
	22.00	4.15	338	8.43	320	5.07	710	1.8	WSW
	23.00	0.41	338	5.64	321	4.96	850	2.7	SSW
Nov 8	0.00	1.28	336	-2.88	321	4.31	892	2.2	WSW
	1.00	0.83	335	-11.28	321	3.80	976	1.3	SSW
	2.00	1.31	334	-5.62	321	3.40	974	2.2	SSW
	3.00	0.07	334	4.08	320	3.03	962	4.5	SSW
	4.00	0.49	335	-3.70	320	2.67	976	2.2	SW
	5.00	0.39	335	-5.05	320	2.45	922	2.2	SSW
	6.00	0.09	335	2.76	321	2.36	928	1.8	S
	7.00	0.07	333	5.80	322	2.27	934	5.4	S
	8.00	0.07	333	7.98	320	2.18	924	3.1	SW
	9.00	0.35	333	5.95	320	2.08	582	4.0	SW
	10.00	1.51	333	2.65	321	2.04	926	5.4	SW
	11.00	0.45	333	2.61	319	2.01	818	6.3	SW
	12.00	0.55	333	2.43	320	1.97	838	6.3	SW
	13.00	1.18	331	-4.72	319	1.95	844	6.7	SW
	14.00	1.82	332	5.39	319	1.97	690	7.2	W
	15.00	2.37	336	3.78	320	2.05	814	8.5	W
	16.00	-2.83	336	-2.99	318	2.14	702	8.5	W
	17.00	-2.15	332	3.22	317	2.21	794	7.6	WNW
	18.00	-3.23	332	4.49	318	2.15	804	7.6	WNW
	19.00	-2.76	334	7.49	328	2.09	814	6.7	WNW

Note: Negative current indicates flow out of harbour.

TABLE 2: Average wind speeds encountered during 72-hour period modelled

Date	Time	Av. direction (from)	Av. Speed (m/sec)
Nov 7	13.00 - 16.00	SSE	3.2
Nov. 7-8	19.00 - 6.00	SW	3
Nov. 8	7.00 - 13.00	SW	5.3
Nov. 8	14.00 - 19.00	W	7.7
Nov. 8-9	20.00 - 1.00	W	2
Nov. 9	2.00 - 8.00	NE	2.4
Nov. 9-10	9.00 - 8.00	E	6.3
Nov. 10	9.00 - 13.00	S	8.1

TABLE 3: Model concentrations for 2 hours at various concentration time step lengths.

(a) Central deep location (about 10 m deep)

Concentration time step/hydrodynamic time step			
10X	5X	3X	1X
0.2099	0.2098	0.2098	0.2096
0.2097	0.2094	0.2096	0.2096
0.2101	0.2099	0.2098	0.2096
0.2102	0.2099	0.2098	0.2096
0.2102	0.2098	0.2098	0.2096
0.2104	0.2098	0.2097	0.2096
0.2105	0.2098	0.2097	0.2096
0.2102	0.2097	0.2097	0.2096
0.2105	0.2095	0.2095	0.2096
0.2105	0.2097	0.2098	0.2096
0.2103	0.2096	0.2096	0.2096
0.2100	0.2097	0.2097	0.2096
0.2101	0.2097	0.2097	0.2096
0.2099	0.2097	0.2097	0.2096
0.2099	0.2100	0.2099	0.2096
0.2095	0.2095	0.2095	0.2096
0.2099	0.2098	0.2097	0.2096
0.2094	0.2096	0.2095	0.2096
0.2094	0.2097	0.2097	0.2095
0.2094	0.2095	0.0295	0.2095
0.2093	0.2097	0.2097	0.2095
0.2091	0.2098	0.2098	0.2095
0.2089	0.2098	0.2098	0.2095
0.2089	0.2098	0.2096	0.2095
0.2088	0.2099	0.2098	0.2095
0.2087	0.2098	0.2097	0.2095
0.2088	0.2097	0.2097	0.2095
0.2089	0.2096	0.2095	0.2095
0.2088	0.2096	0.2095	0.2095
0.2091	0.2098	0.2098	0.2095
0.2089	0.2095	0.2096	0.2095
0.2092	0.2097	0.2097	0.2095
0.2089	0.2096	0.2096	0.2095
0.2091	0.2097	0.2096	0.2095
0.2090	0.2095	0.2095	0.2095

Note: All concentrations are in kg/m^3 (mg/l÷1000), and printed at an interval of 200 sec. Hydrodynamic time step is 20 sec.

TABLE 3: continued

Model concentrations for 2 hours at various concentration time step lengths.

(b) Southern shallow location (about 2 m deep)

Concentration time step/hydrodynamic time step			
10X	5X	3X	1X
0.2093	0.2099	0.2102	0.2099
0.2097	0.2098	0.2096	0.2099
0.2091	0.2098	0.2099	0.2099
0.2087	0.2101	0.2103	0.2099
0.2093	0.2097	0.2093	0.2099
0.2096	0.2109	0.2107	0.2099
0.2082	0.2091	0.2090	0.2099
0.2099	0.2018	0.2105	0.2099
0.2088	0.2098	0.2097	0.2099
0.2086	0.2100	0.2098	0.2099
0.2086	0.2105	0.2101	0.2099
0.2066	0.2095	0.2093	0.2099
0.2087	0.2112	0.2103	0.2099
0.2051	0.2097	0.2101	0.2099
0.2066	0.2109	0.2100	0.2099
0.2063	0.2106	0.2102	0.2099
0.2059	0.2104	0.2097	0.2099
0.2060	0.2105	0.2098	0.2099
0.2064	0.2107	0.2101	0.2099
0.2063	0.2097	0.2092	0.2099
0.2079	0.2112	0.2106	0.2099
0.2062	0.2093	0.2089	0.2099
0.2086	0.2111	0.2104	0.2099
0.2082	0.2099	0.2096	0.2099
0.2089	0.2102	0.2099	0.2099
0.2108	0.2107	0.2099	0.2099
0.2101	0.2092	0.2093	0.2099
0.2128	0.2106	0.2100	0.2099
0.2125	0.2095	0.2099	0.2099
0.2144	0.2097	0.2096	0.2099
0.2143	0.2097	0.2101	0.2099
0.2155	0.2096	0.2094	0.2099
0.2156	0.2095	0.2097	0.2098
0.2156	0.2095	0.2099	0.2098
0.2154	0.2086	0.2089	0.2098
0.2171	0.2103	0.2104	0.2098

Note: All concentrations are in kg/m^3 (mg/l÷1000), and printed at an interval of 200 sec. Hydrodynamic time step length is 20 sec.

TABLE 3: continued

Model concentrations for 2 hours at various concentration time step lengths.

(c) Southwestern location (about 8 m deep but close to shore)

Concentration time step/hydrodynamic time step			
10X	5X	3X	1X
0.2097	0.2099	0.2100	0.2097
0.2090	0.2092	0.2094	0.2097
0.2096	0.2100	0.2100	0.2097
0.2091	0.2094	0.2096	0.2097
0.2094	0.2094	0.2095	0.2097
0.2093	0.2095	0.2097	0.2097
0.2092	0.2093	0.2096	0.2097
0.2098	0.2098	0.2098	0.2097
0.2089	0.2093	0.2096	0.2097
0.2091	0.2094	0.2095	0.2097
0.2097	0.2098	0.2099	0.2097
0.2093	0.2092	0.2095	0.2096
0.2097	0.2094	0.2096	0.2096
0.2092	0.2091	0.2096	0.2096
0.2098	0.2092	0.2096	0.2096
0.2100	0.2094	0.2098	0.2096
0.2093	0.2087	0.2094	0.2096
0.2104	0.2095	0.2099	0.2096
0.2099	0.2092	0.2097	0.2096
0.2101	0.2092	0.2095	0.2096
0.2102	0.2093	0.2098	0.2096
0.2100	0.2090	0.2095	0.2096
0.2108	0.2095	0.2096	0.2096
0.2102	0.2089	0.2095	0.2096
0.2102	0.2089	0.2094	0.2096
0.2105	0.2094	0.2099	0.2096
0.2101	0.2090	0.2095	0.2096
0.2105	0.2093	0.2098	0.2096
0.2101	0.2092	0.2097	0.2096
0.2103	0.2093	0.2096	0.2096
0.2108	0.2095	0.2099	0.2096
0.2100	0.2088	0.2093	0.2096
0.2106	0.2094	0.2097	0.2096
0.2104	0.2090	0.2094	0.2096
0.2104	0.2092	0.2093	0.2095
0.2100	0.2092	0.2095	0.2095

Note: All concentrations are in kg/m^3 (mg/l÷1000), and printed at an interval of 200 sec. Hydrodynamic time step length is 20 sec.

TABLE 4: Toronto Harbour Oscillation Periods as Determined by Numerical Model and Merian Equation

(a) Longitudinal

n	Model Period (min)	Merian Eqn. Period (min)
1	12.8	12.0
2	8.4	6.9
3	4.7	4.9

(b) Transverse

n	Model Period (min)	Merian Eqn. Period (min)
1	6.3	6.4
2	4.3	3.7

Note: Model periods are obtained from spectral analysis of model u and v velocities.

Merian equation periods are calculated from the equation for a symmetrical concave parabolic basin (Hutchinson, 1957, p. 304).

TABLE 5: Significant Coherences between Model Data and Field Data, Toronto Harbour

Data	Model Location	Meter Location	Significant Periods, hr
U velocity	Average of A, C	129	1.8-5.3
V velocity	D	129	1.1-2.0 5.3-16.0
Concentration (Dissolved solids)	G	128	1.4-1.7 2.2-2.5 6.7 20.0

Note: Components of current meter velocity parallel to x and y axes of model grid (i.e. east and west gaps) were used in cross-correlation with model U and V velocities, respectively.

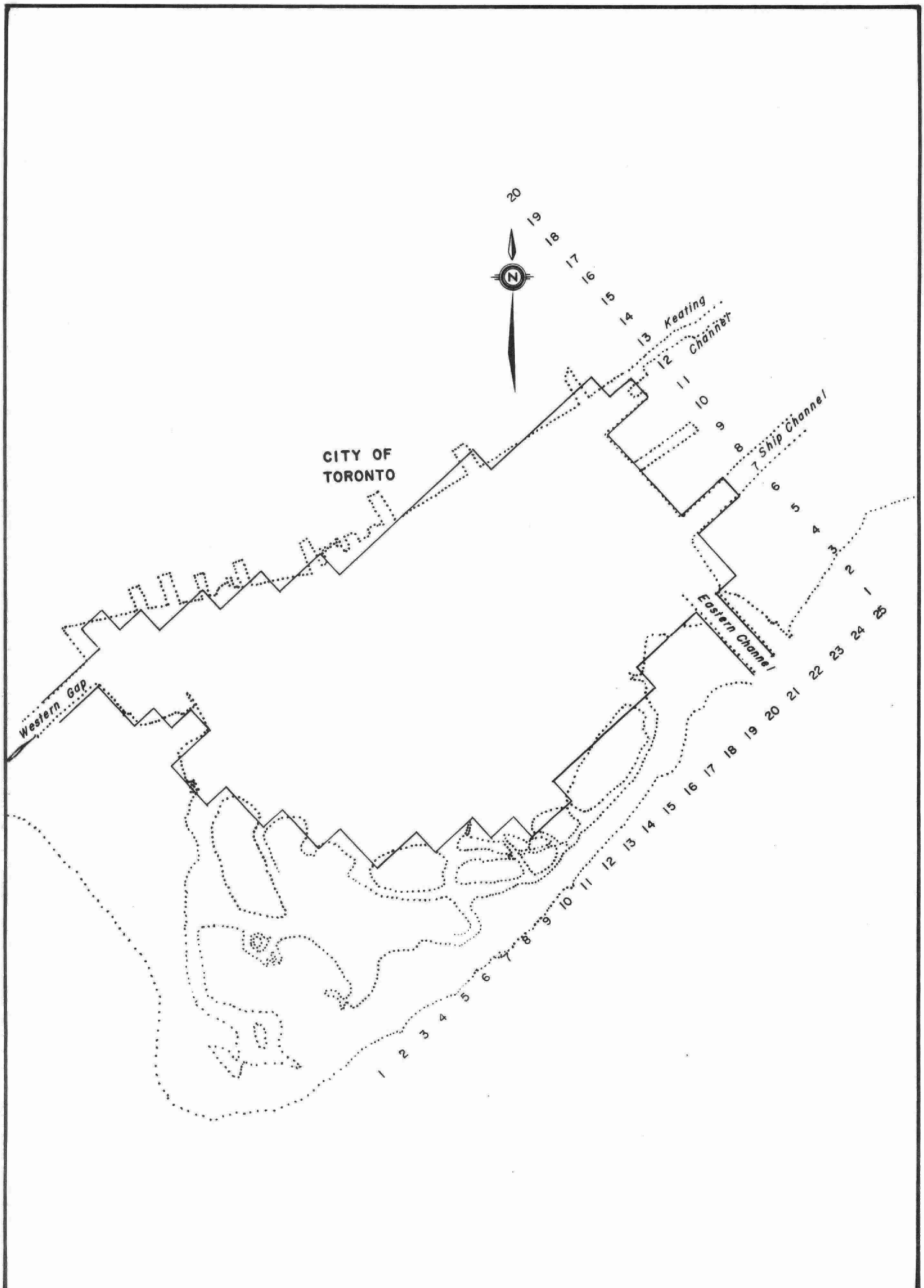


FIGURE 1 : TORONTO HARBOUR NUMERICAL MODEL GRID

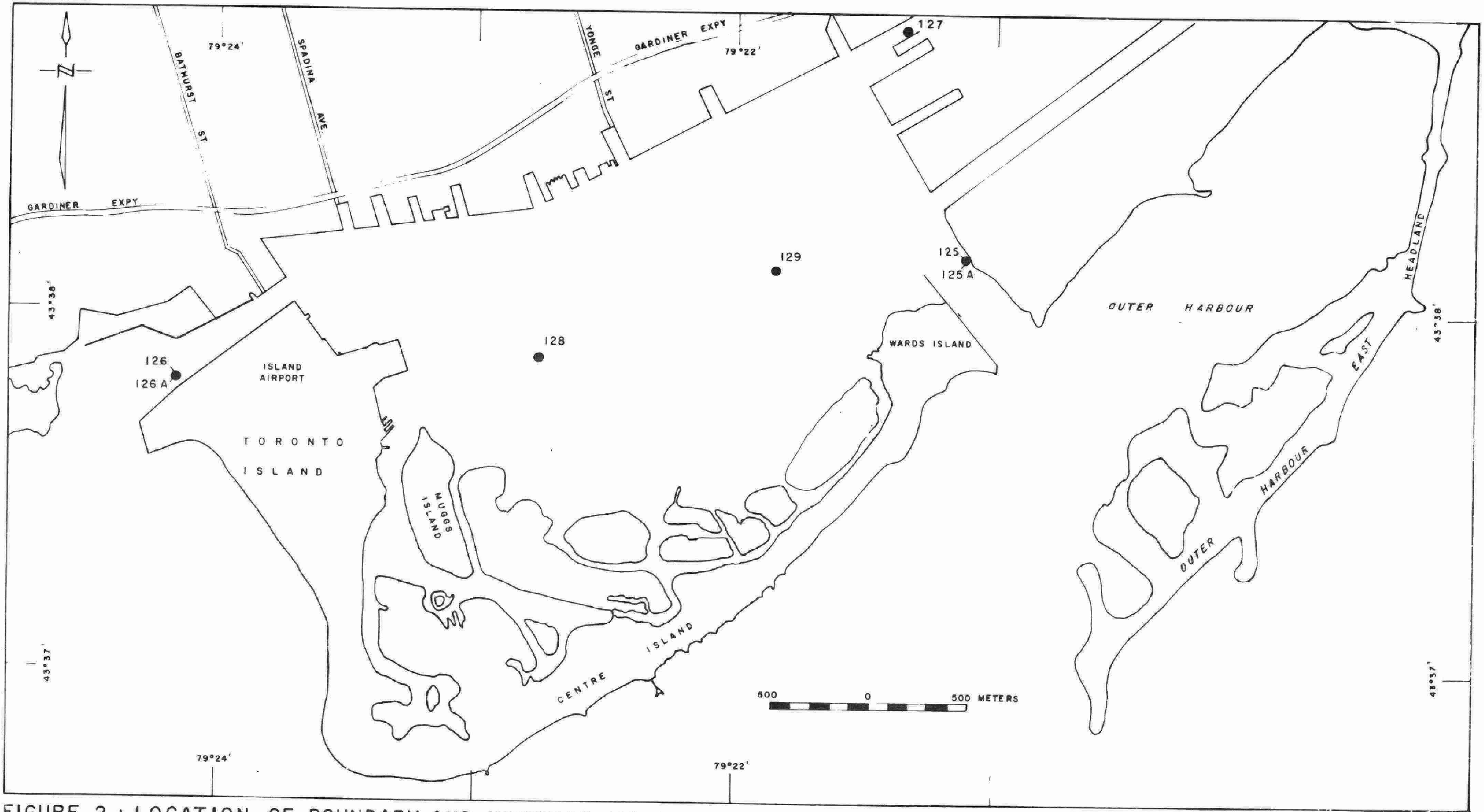


FIGURE 2 : LOCATION OF BOUNDARY AND INTERIOR MONITORING POINTS FOR TORONTO HARBOUR NUMERICAL MODEL

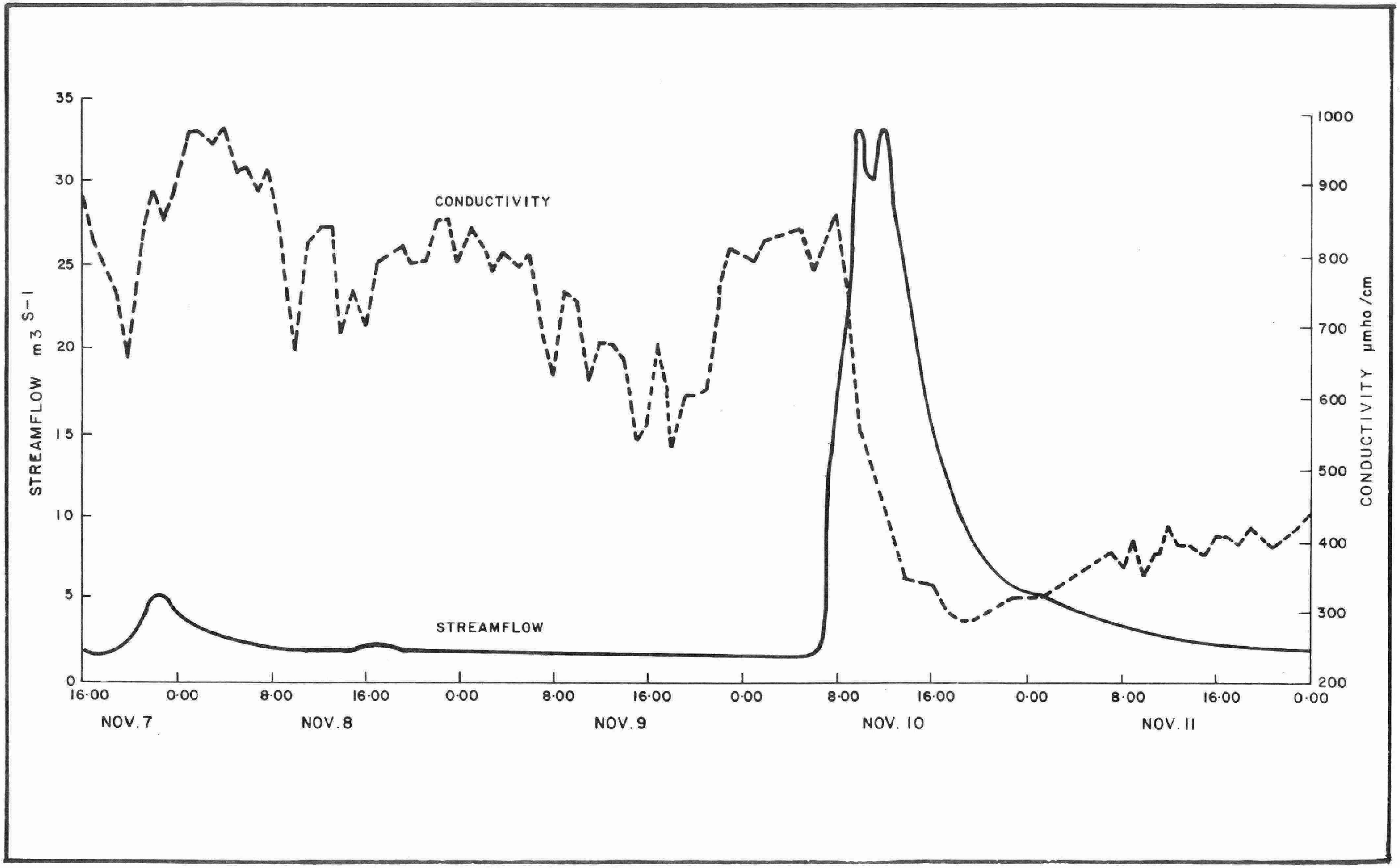


FIGURE 3 : RELATIONSHIP BETWEEN DON RIVER STREAMFLOW AND KEATING CHANNEL CONDUCTIVITY NOV. 7-11, 1975

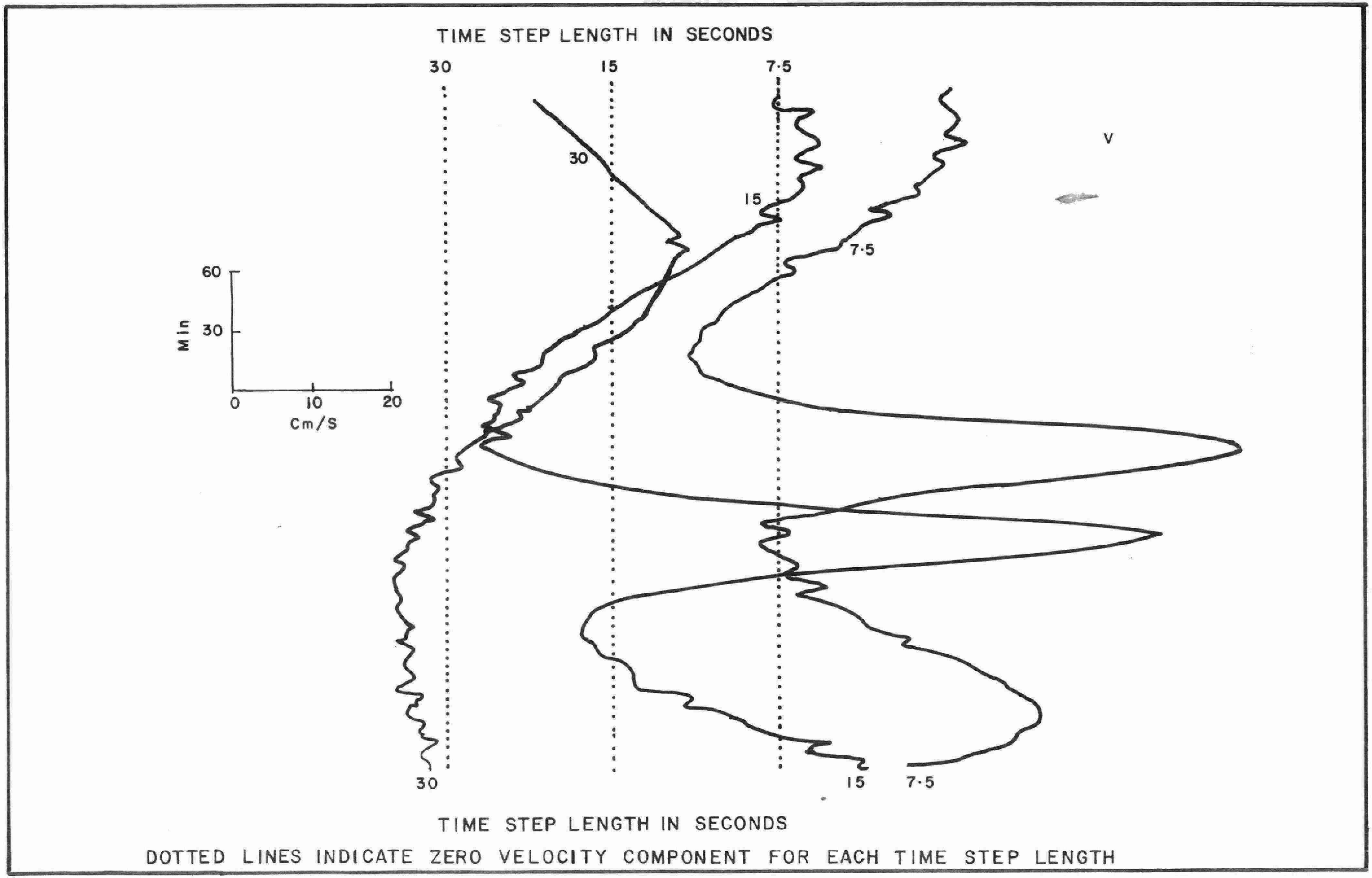


FIGURE 4 : ORIGINAL PREDICTION OF NUMERICAL MODEL CURRENTS AT TIME STEPS OF 30, 15 AND 7·5 SECONDS

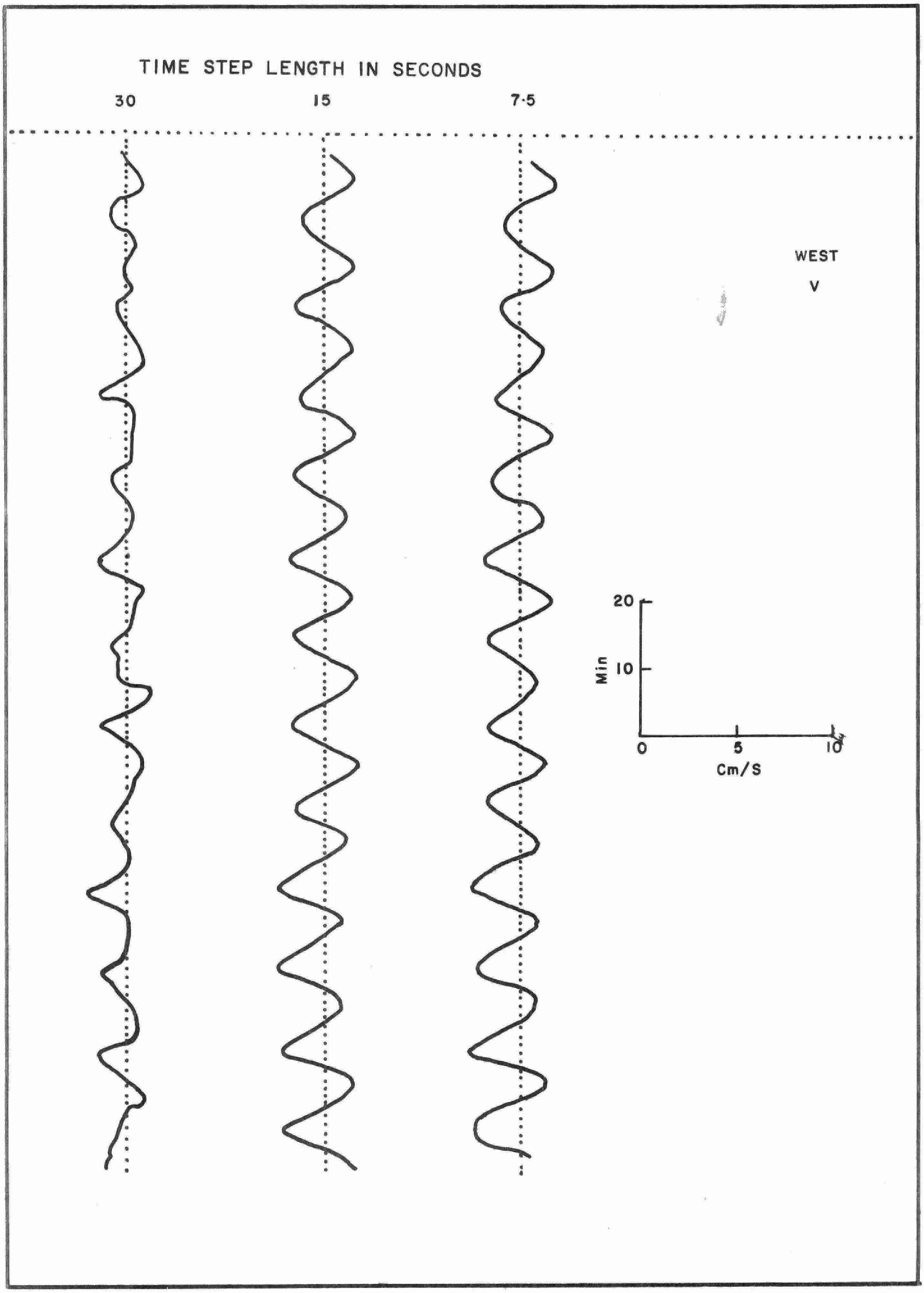


FIGURE 5 : PREDICTIONS OF NUMERICAL MODEL CURRENTS WITH REVISED WATER DEPTHS AT TIME STEPS OF 30, 15 AND 7·5 SECONDS

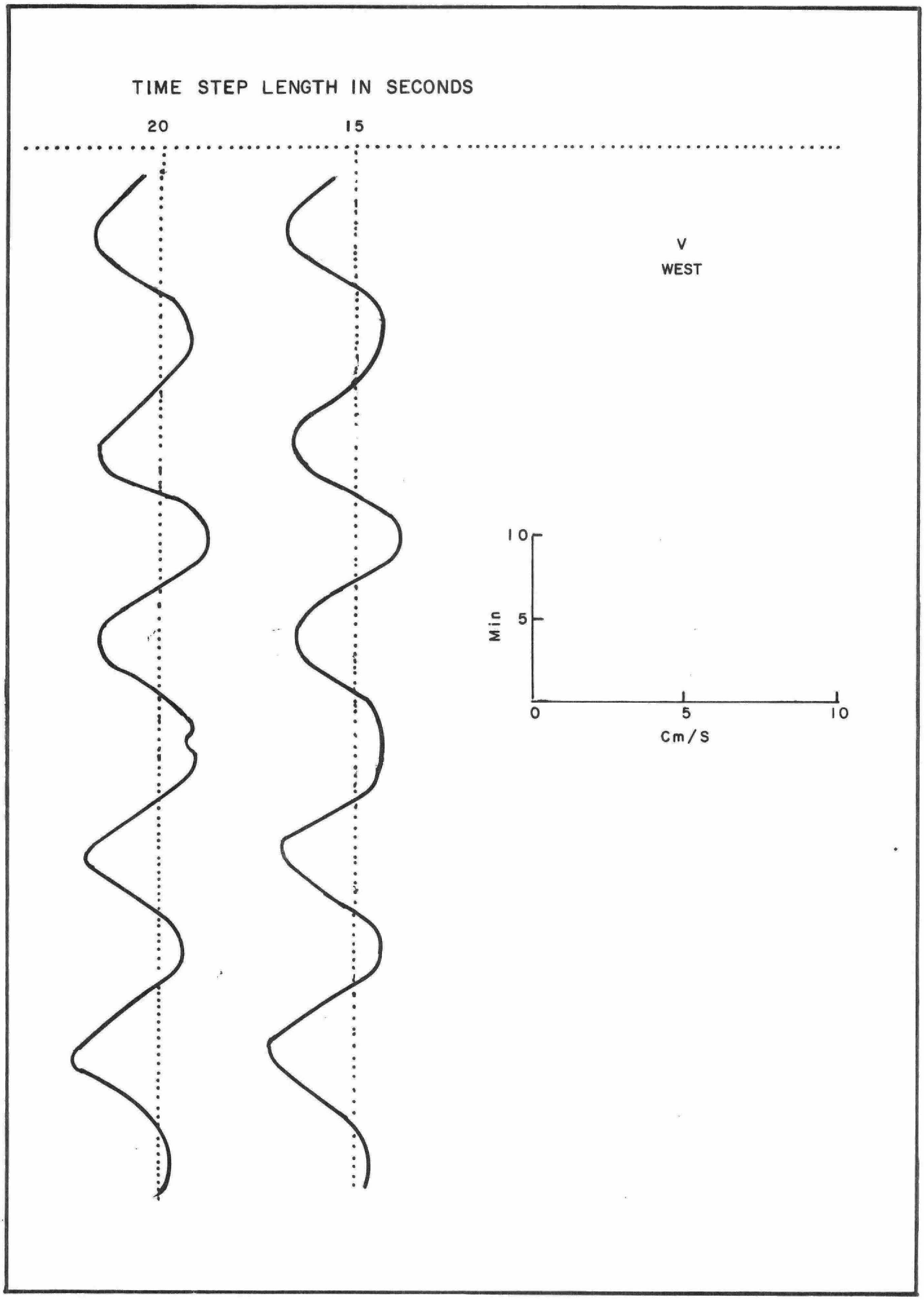


FIGURE 6 : PREDICTIONS OF NUMERICAL MODEL CURRENTS WITH REVISED WATER DEPTHS AT TIME STEPS OF 20 AND 15 SECONDS

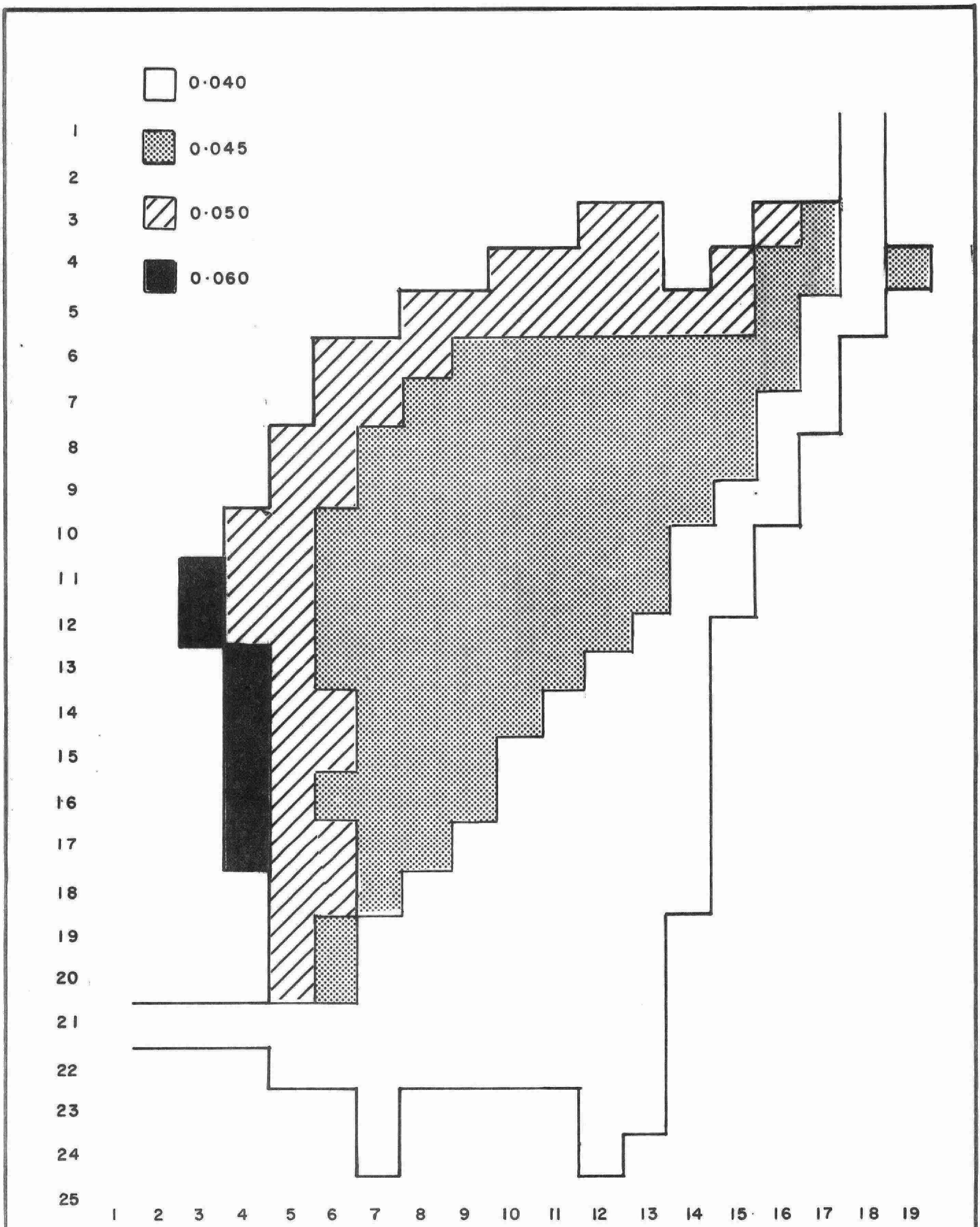


FIGURE 7 : MANNING'S N VALUES USED IN TORONTO HARBOUR MODEL

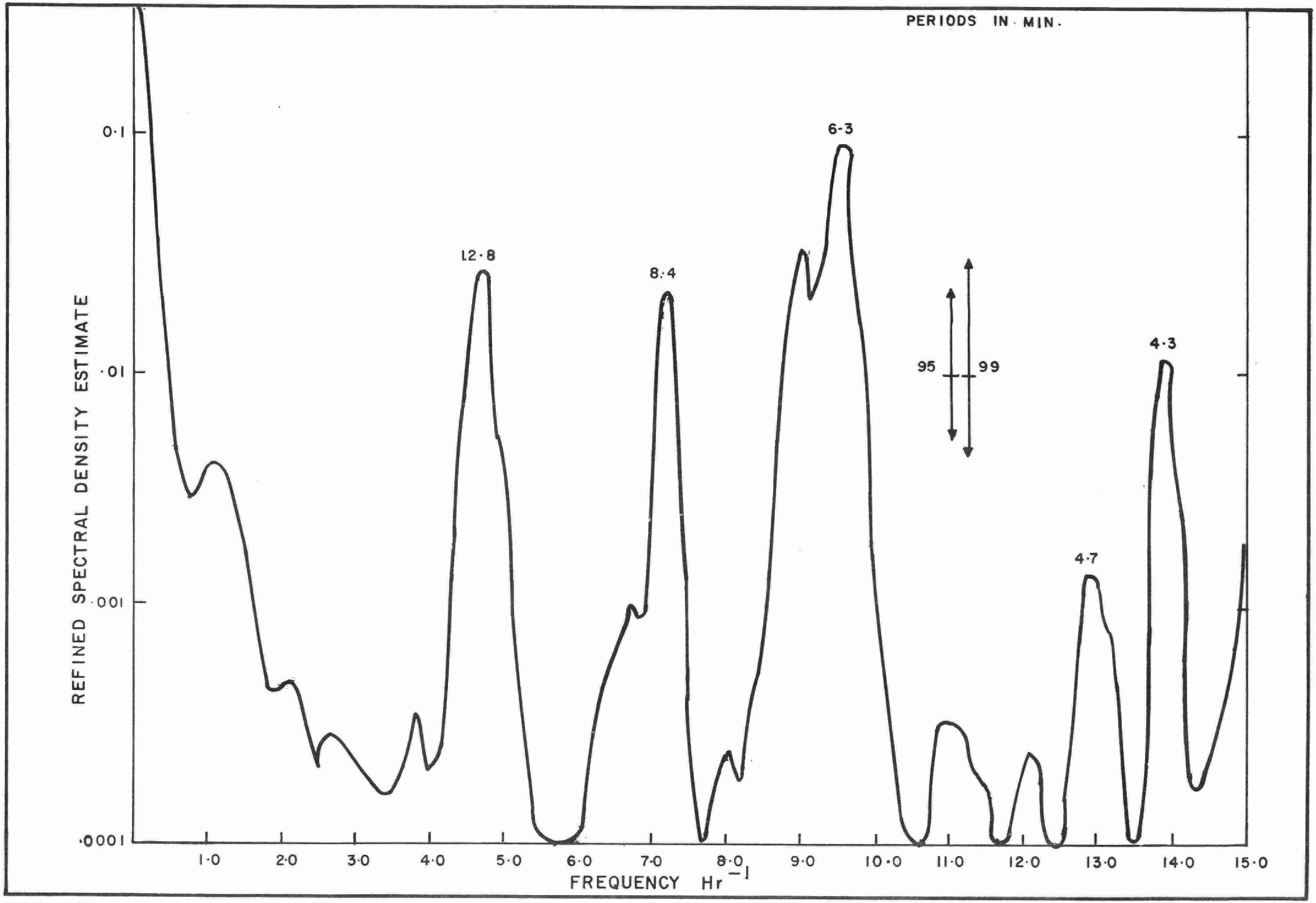


FIGURE 8 : AUTOSPECTRA OF UNSMOOTHED MODEL U VELOCITY AT EAST METER LOCATION (129)

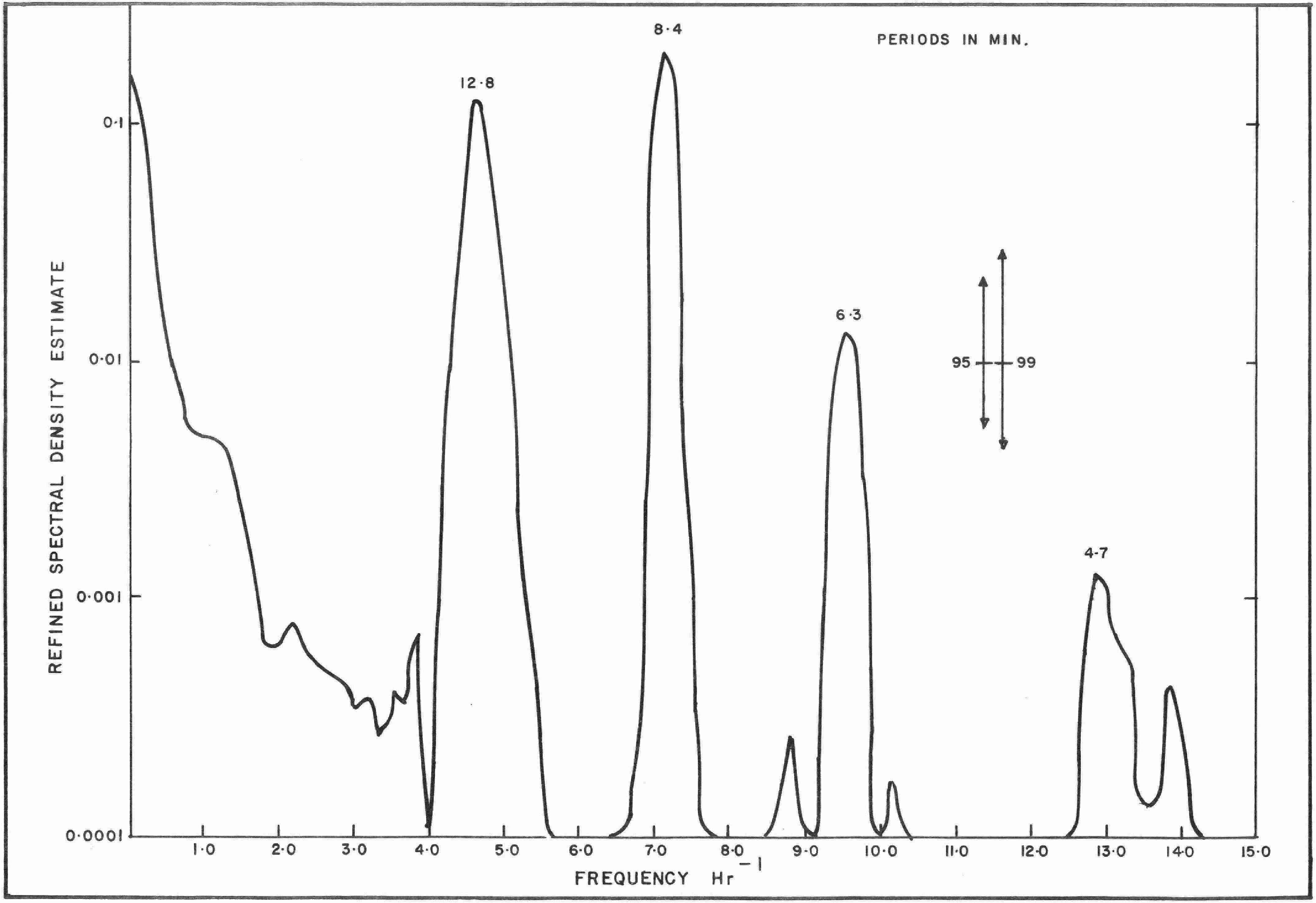


FIGURE 9 : AUTOSPECTRA OF UNSMOOTHED MODEL V VELOCITY AT EAST METER LOCATION (129)

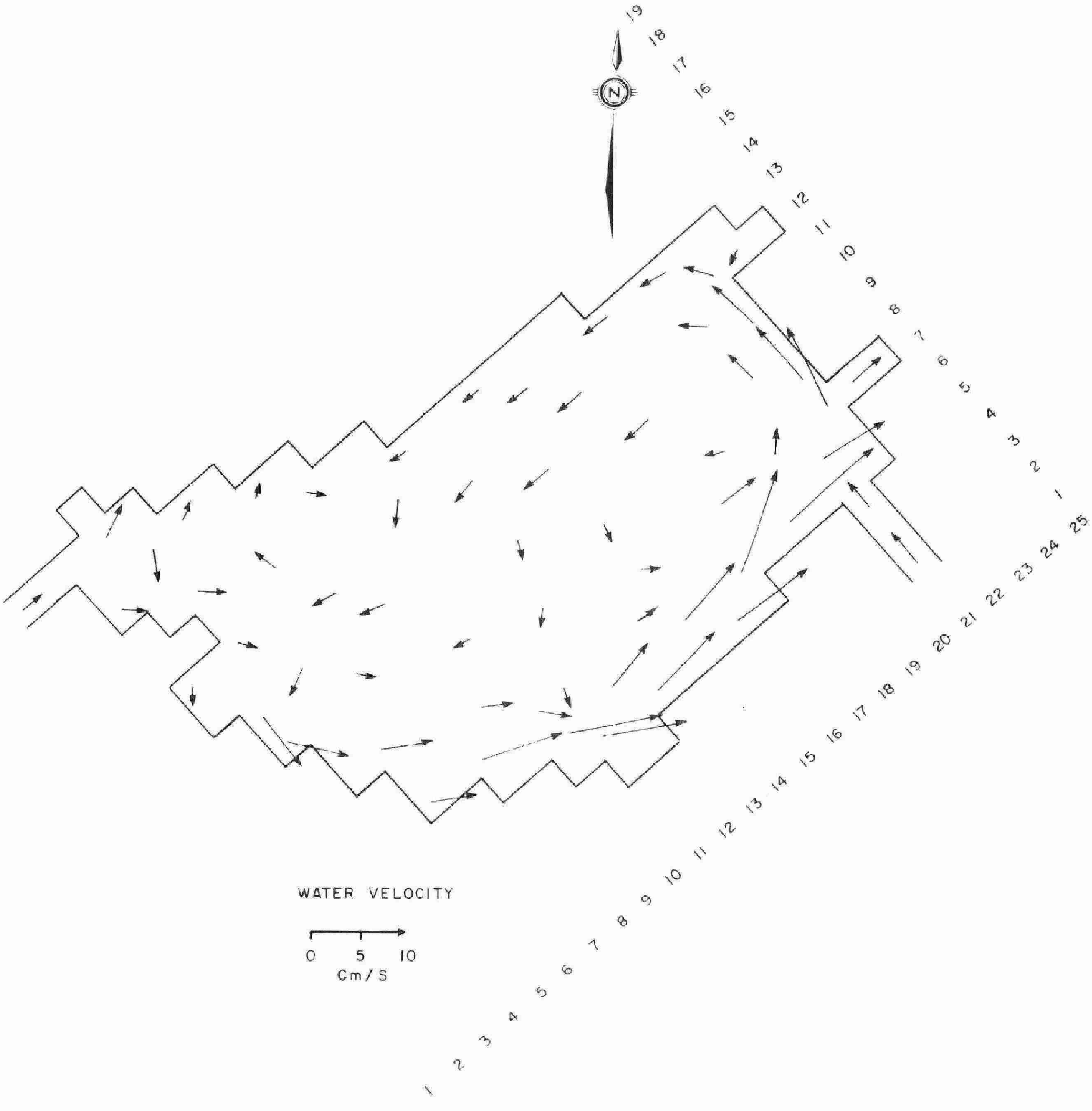


FIGURE 10 a : NUMERAL MODEL CURRENTS AFTER 27 HOURS (WIND PRIMARILY FROM WEST, AVERAGE SPEED 8 M/S)

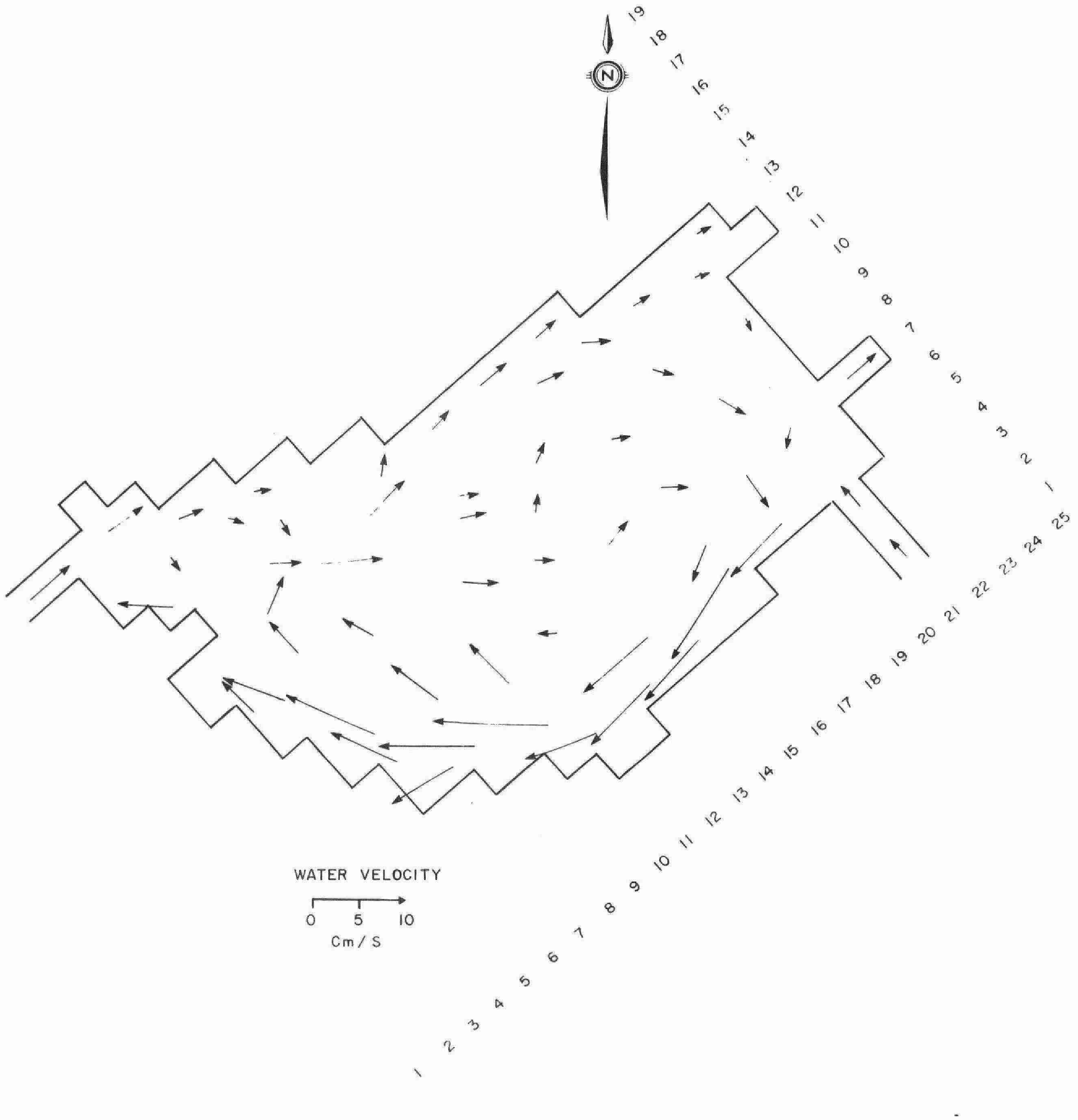


FIGURE 10 b : NUMERICAL MODEL CURRENTS AFTER 60 HOURS (WIND PRIMARILY FROM EAST FOR PREVIOUS 24 HOURS, AVER SPEED 6 M/S)

Figure 11a

Numerical Model Predictions of Total Dissolved Solids
After 24 Hours of Modelled Time

Indicated figures are values of total dissolved
solids (mg/l) for each contour line.

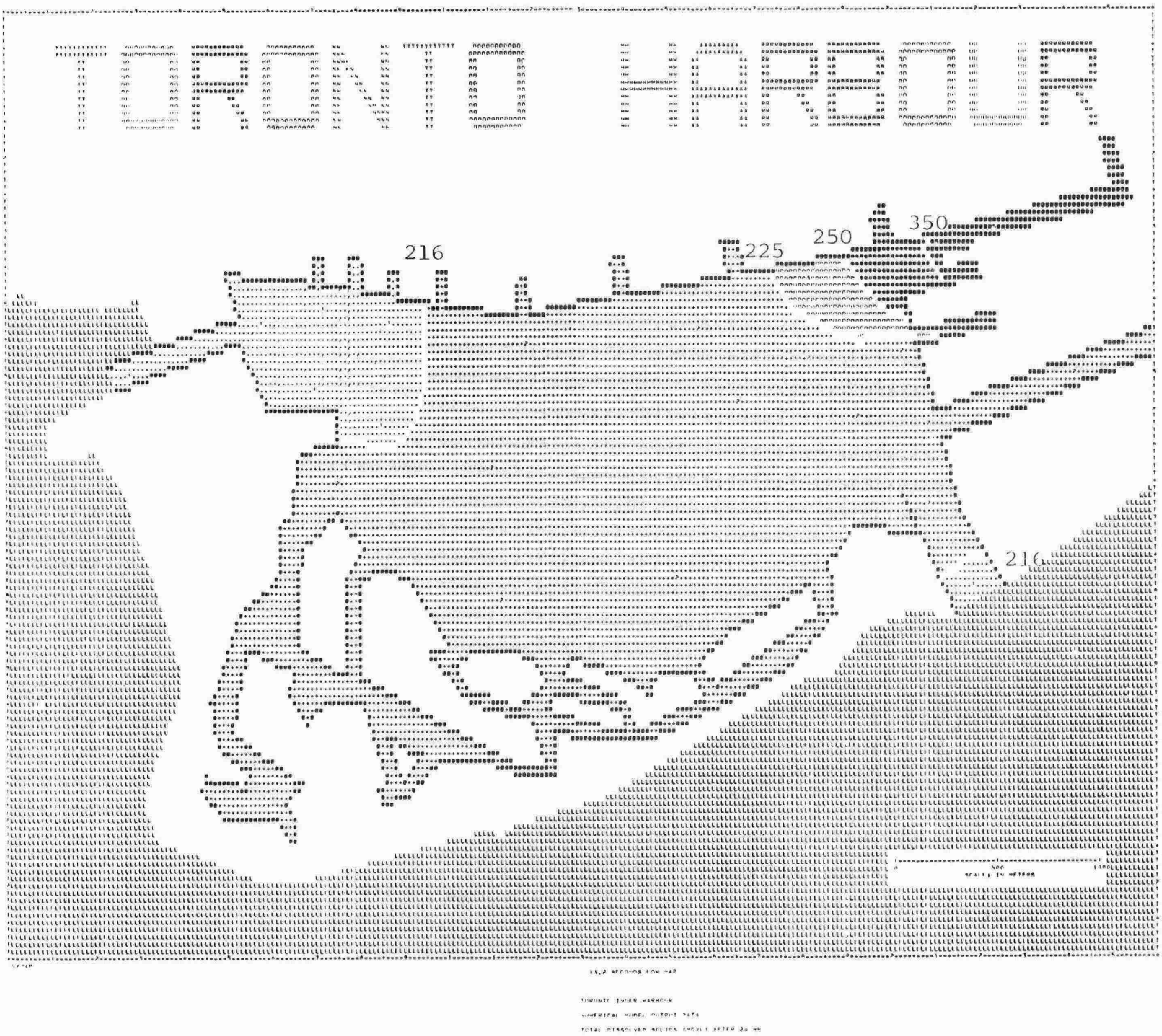


Figure 11b

Numerical Model Predictions of Total Dissolved Solids
After 30 Hours of Modelled Time

Indicated figures are values of total dissolved
solids (mg/l) for each contour line.

Figure 11c

Numerical Model Predictions of Total Dissolved Solids
After 36 Hours of Modelled Time

Indicated figures are values of total dissolved solids (mg/l) for each contour line.

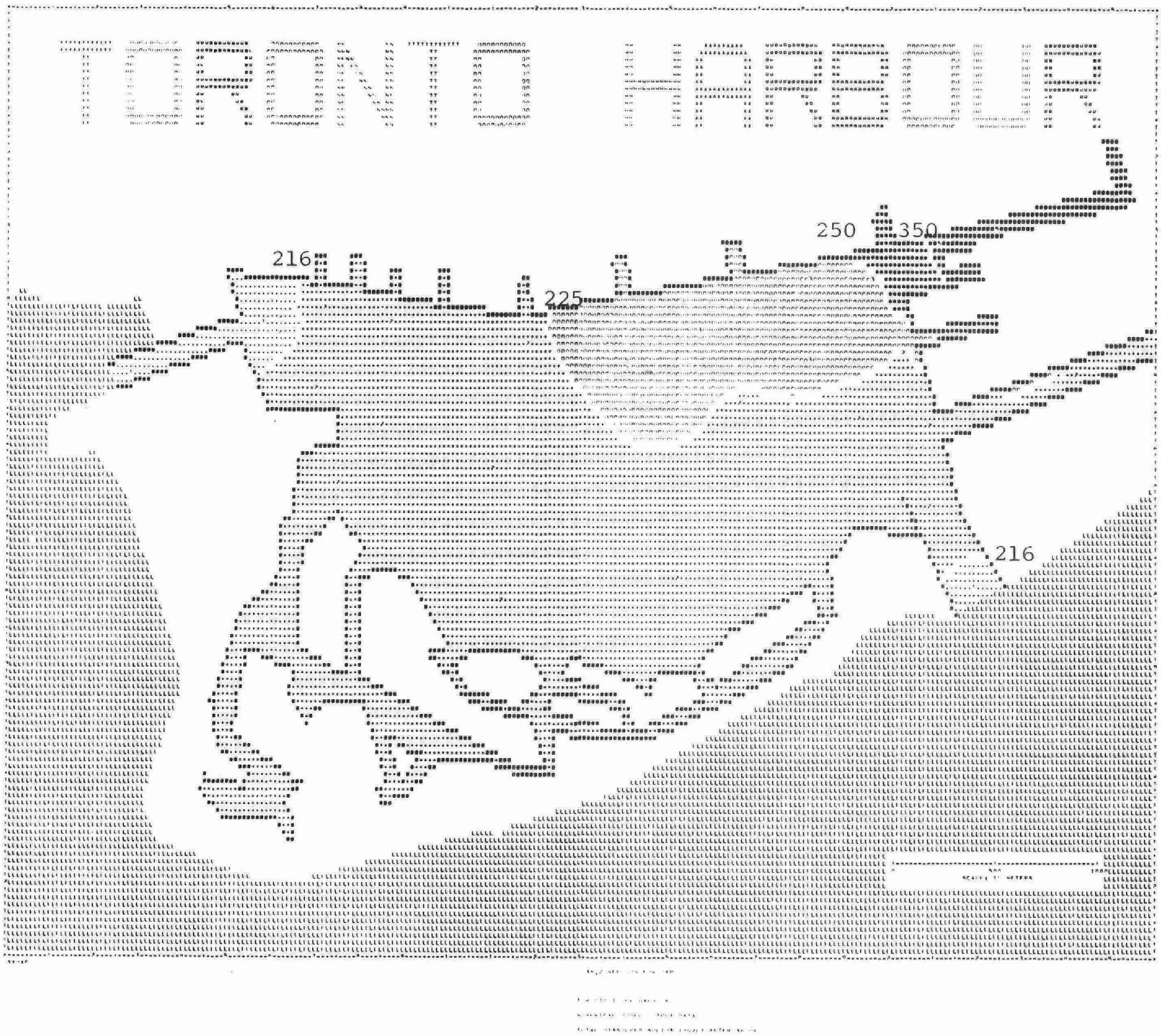


Figure 11d

Numerical Model Predictions of Total Dissolved Solids
After 42 Hours of Modelled Time

Indicated figures are values of total dissolved solids (mg/l) for each contour line.

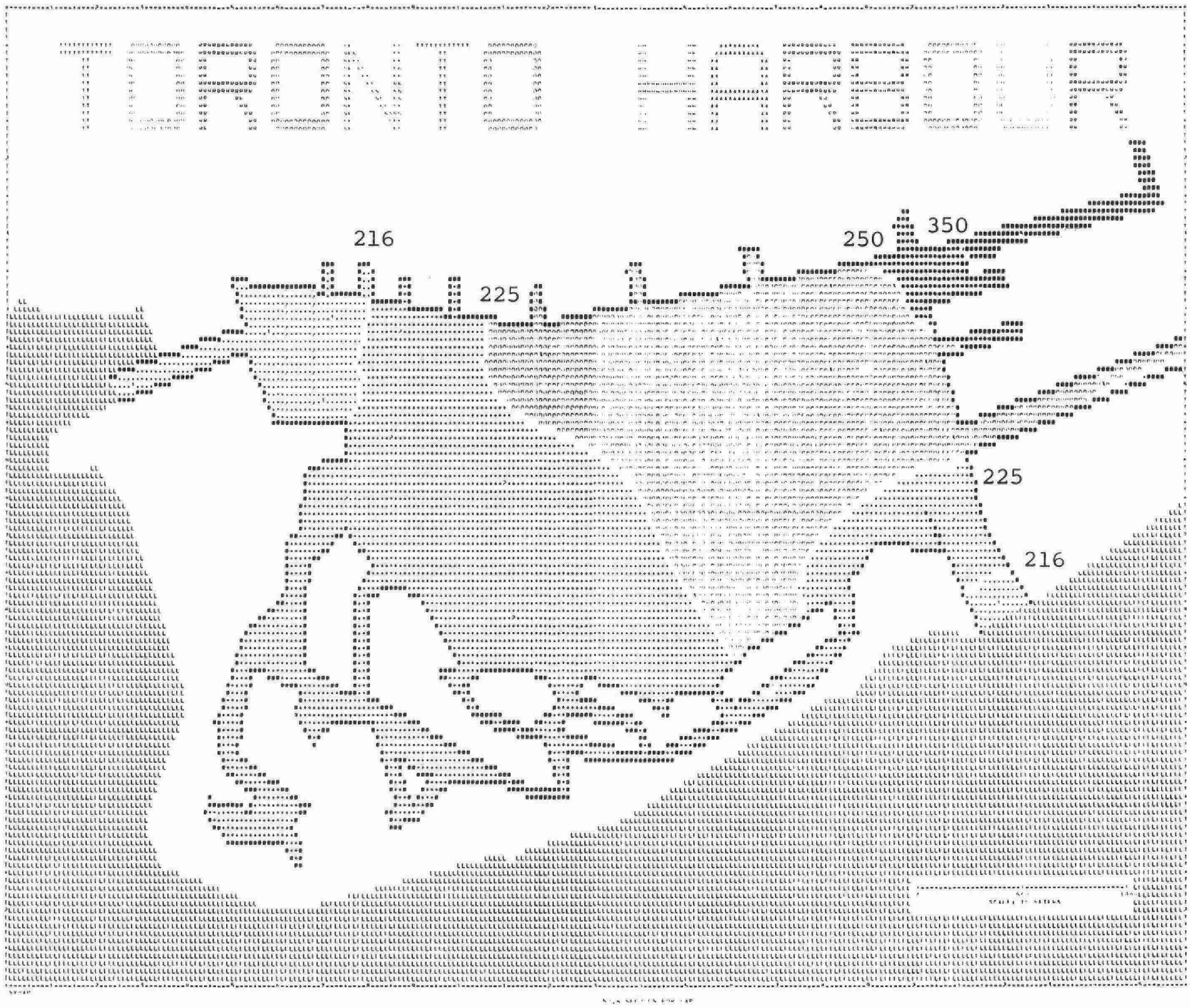


Figure 11e

Numerical Model Predictions of Total Dissolved Solids
After 48 Hours of Modelled Time

Indicated figures are values of total dissolved solids (mg/l) for each contour line.

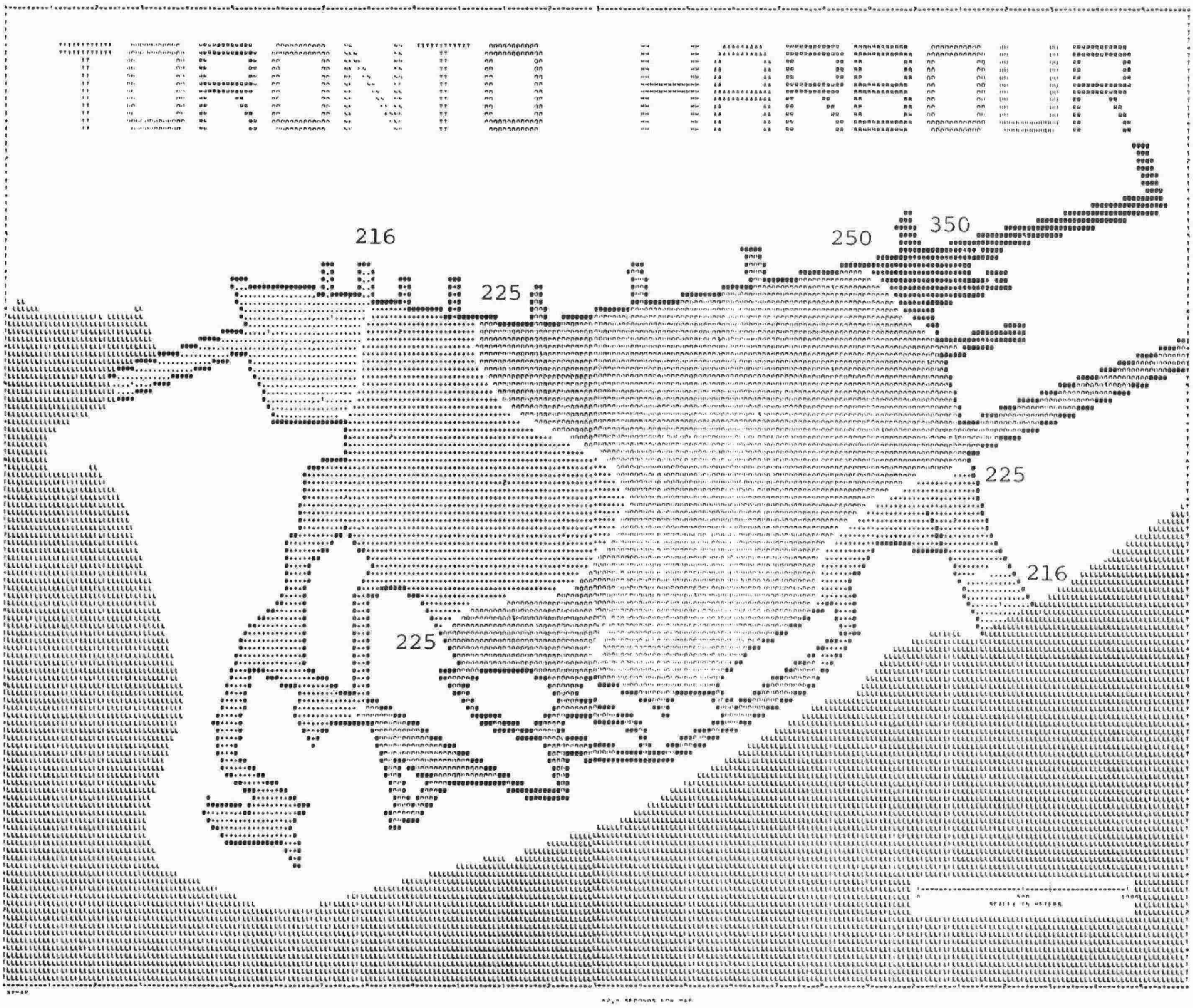


Figure 11f

Numerical Model Predictions of Total Dissolved Solids
After 54 Hours of Modelled Time

Indicated figures are values of total dissolved solids (mg/l) for each contour line.

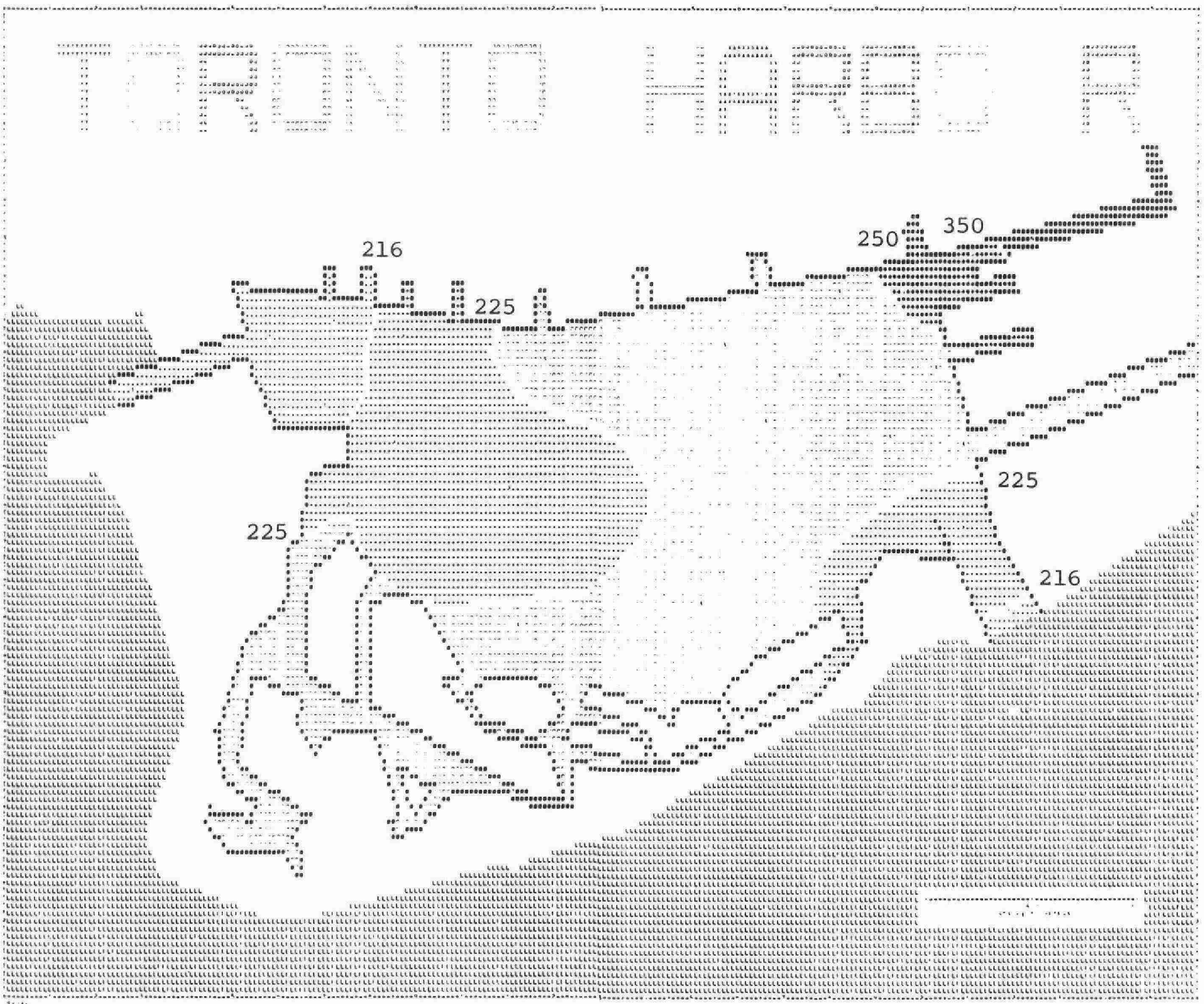


Figure 11g

Numerical Model Predictions of Total Dissolved Solids
After 60 Hours of Modelled Time

Indicated figures are values of total dissolved solids (mg/l) for each contour line.

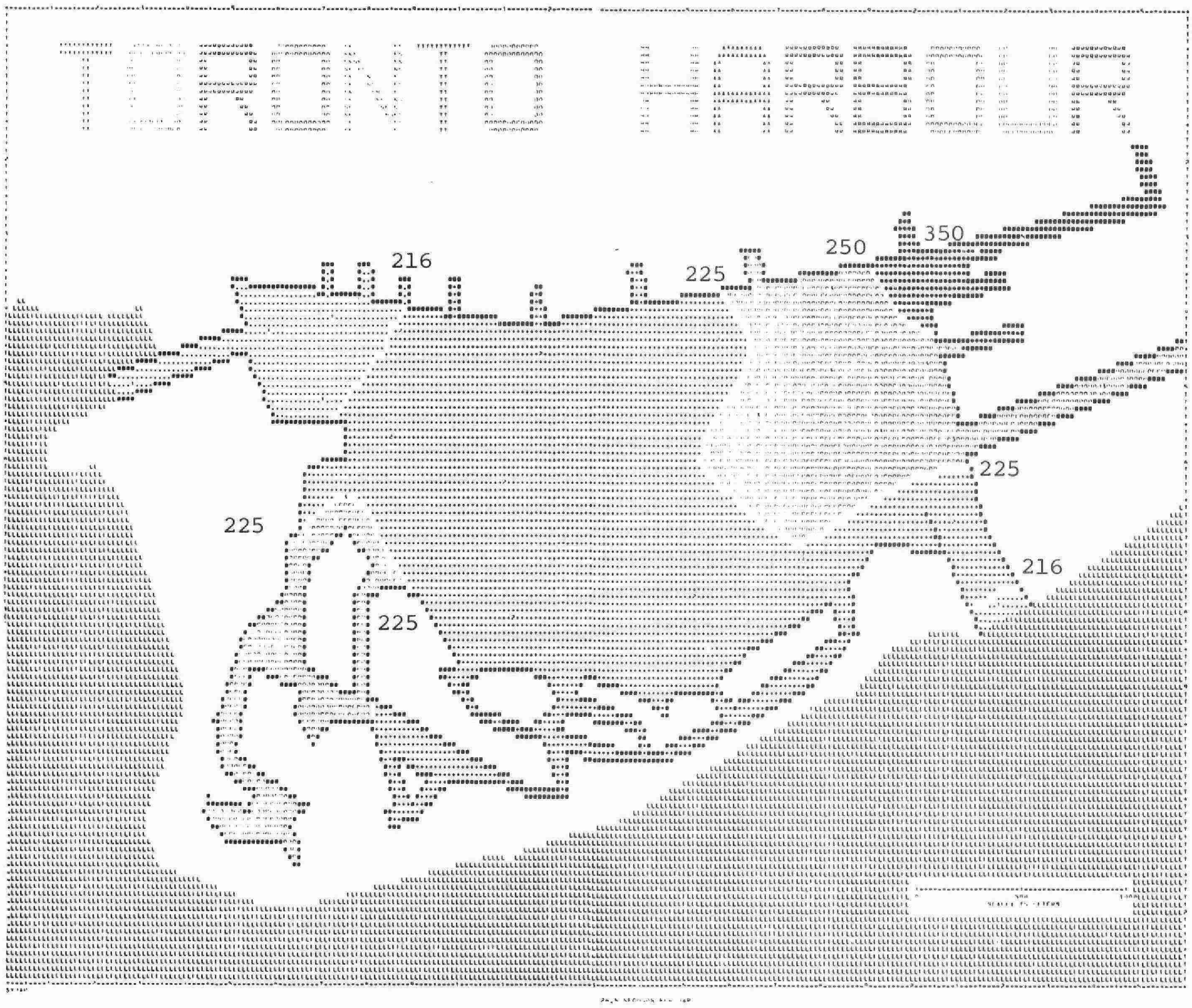


Figure 11h

Numerical Model Predictions of Total Dissolved Solids
After 66 Hours of Modelled Time

Indicated figures are values of total dissolved solids (mg/l) for each contour line.

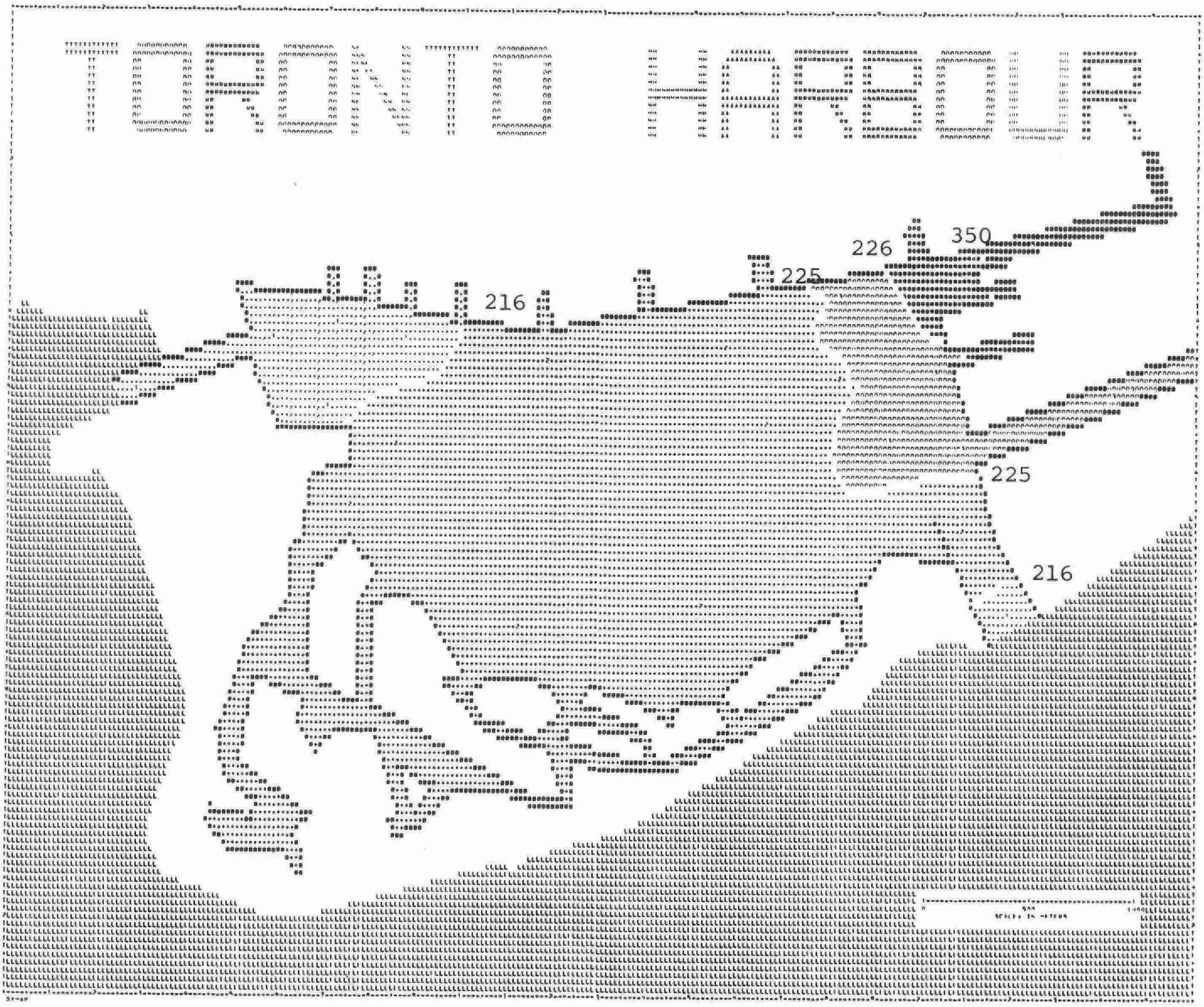
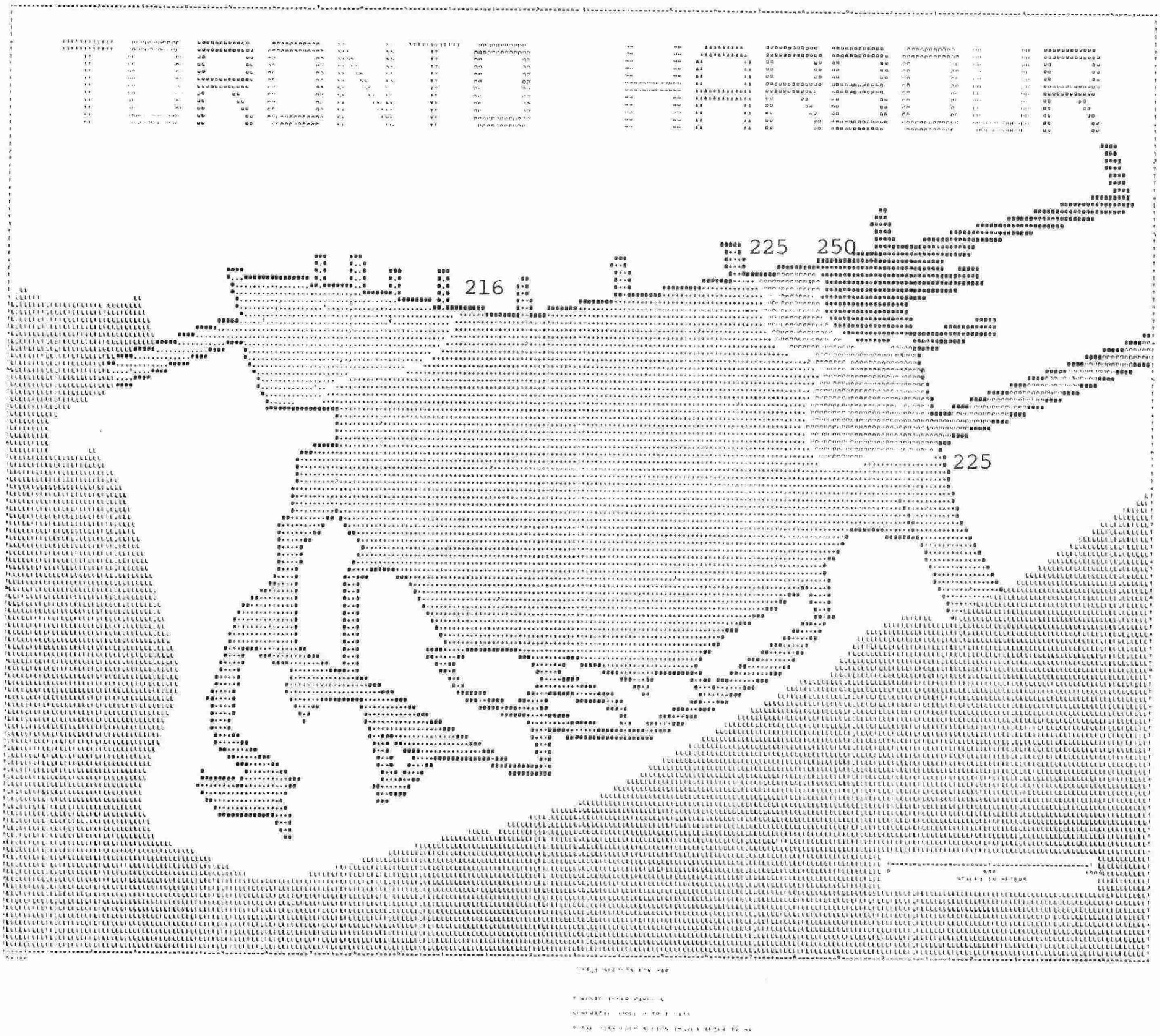


Figure 11i

Numerical Model Predictions of Total Dissolved Solids
After 72 Hours of Modelled Time

Indicated figures are values of total dissolved solids (mg/l) for each contour line.



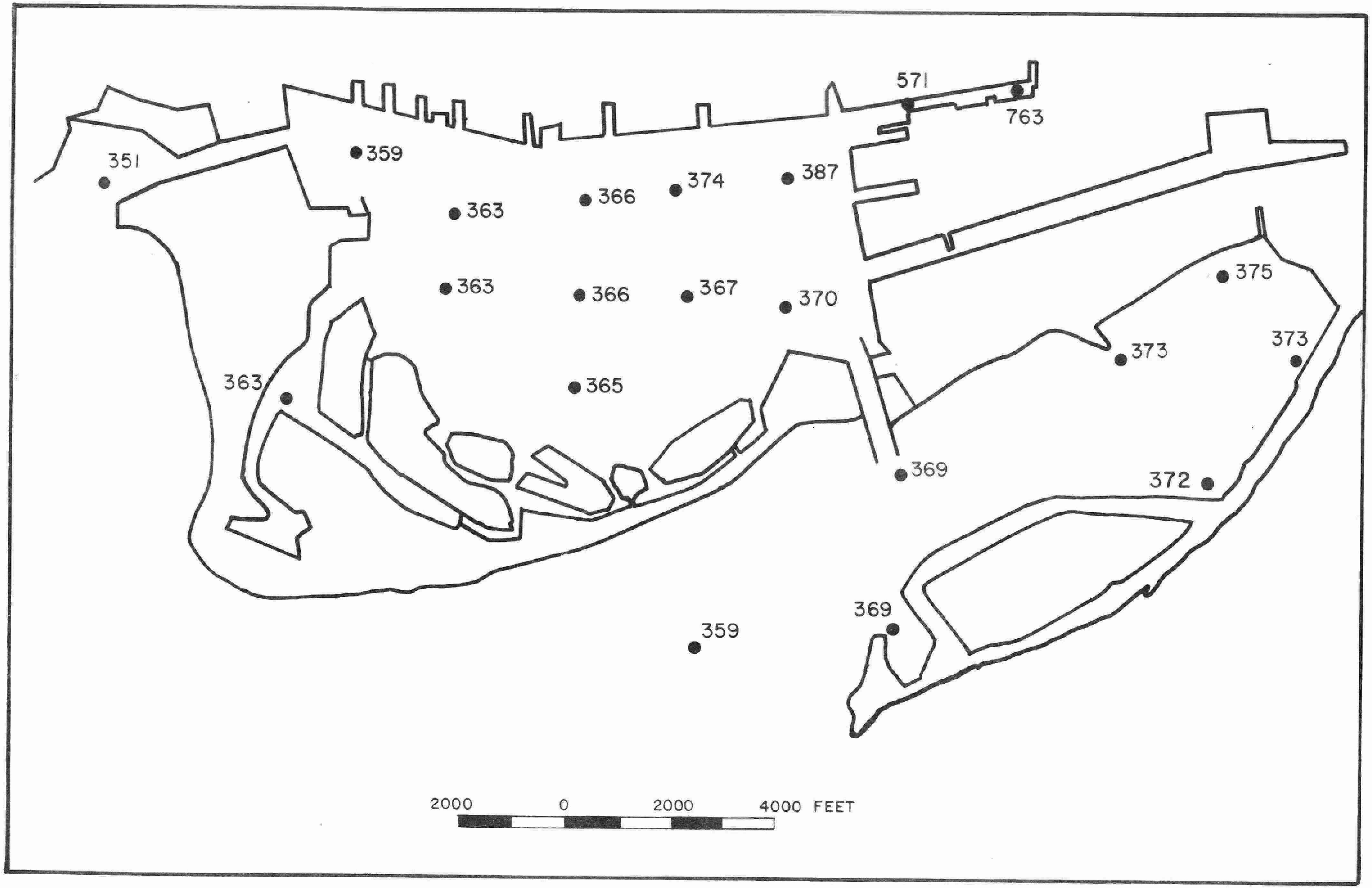


FIGURE 12 : AVERAGE TORONTO HARBOUR CONDUCTIVITIES (μmho) MEASURED IN 1973.

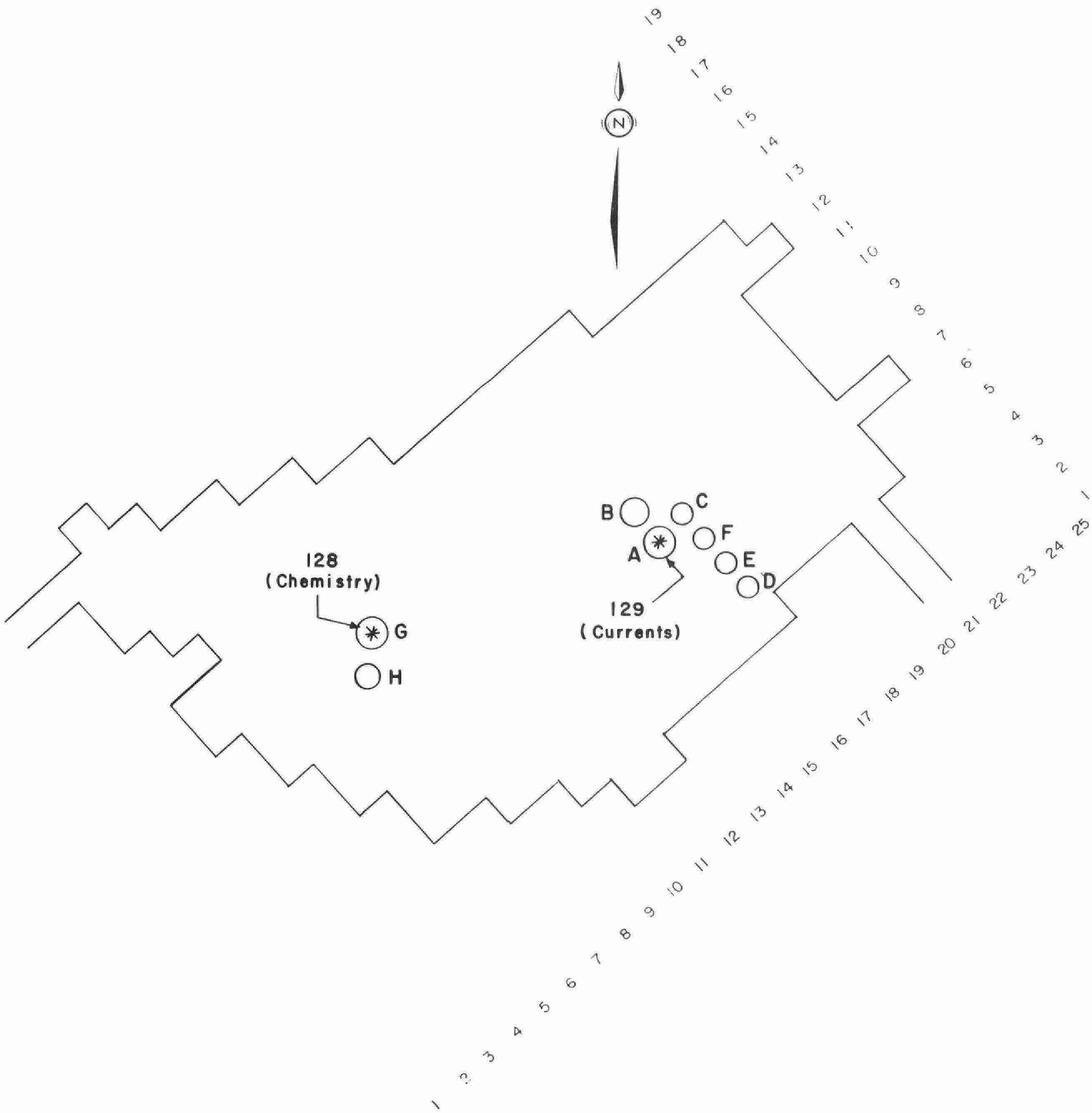


FIGURE 13 : LOCATION OF MODEL DATA USED FOR CROSS CORRELATION

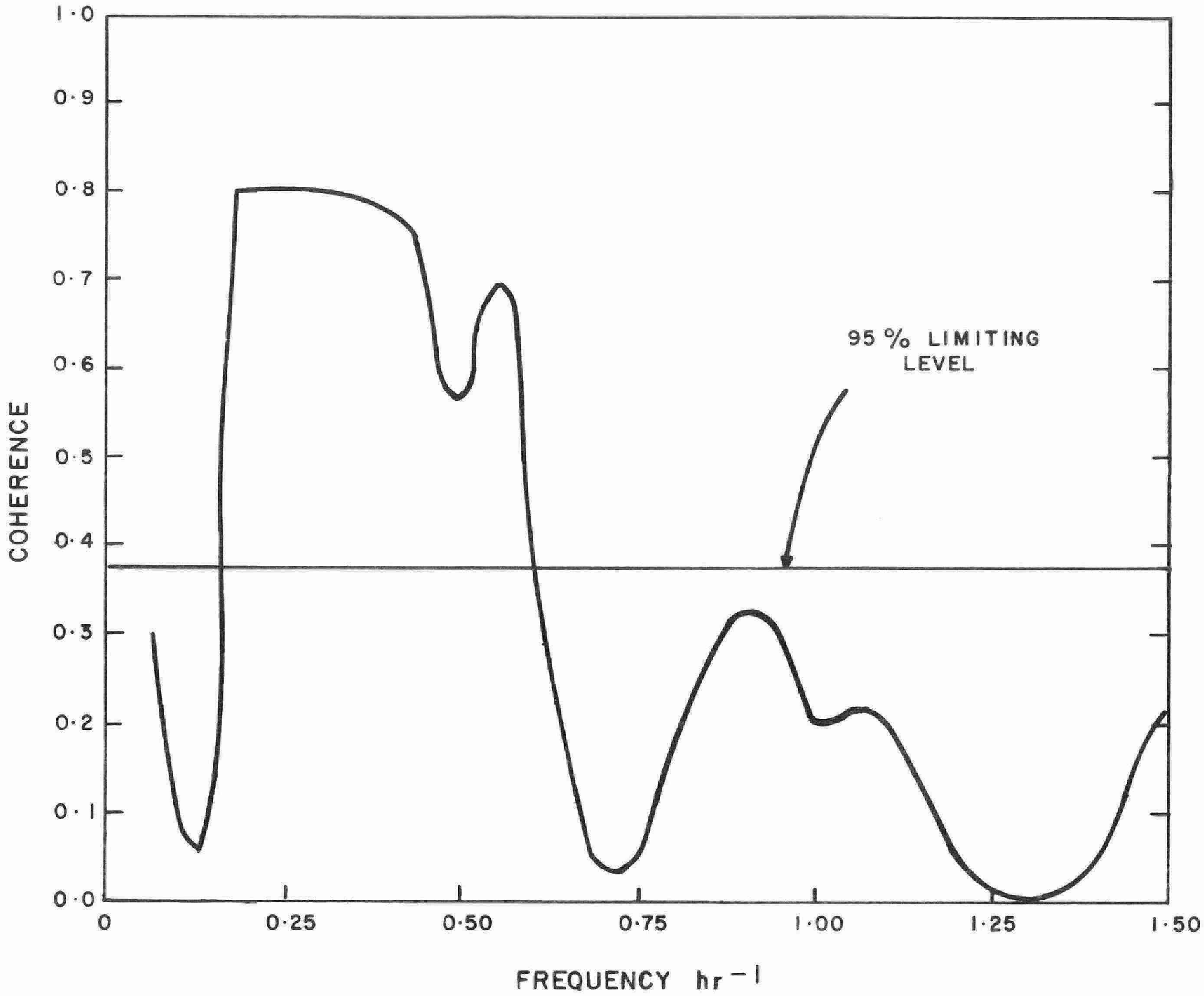


FIGURE 14: CROSS CORRELATION OF MODEL U VELOCITY (AVERAGE OF POINTS A AND C) WITH COMPONENT OF CURRENT METER DATA PARALLEL TO EAST GAP, LOCATION 129.

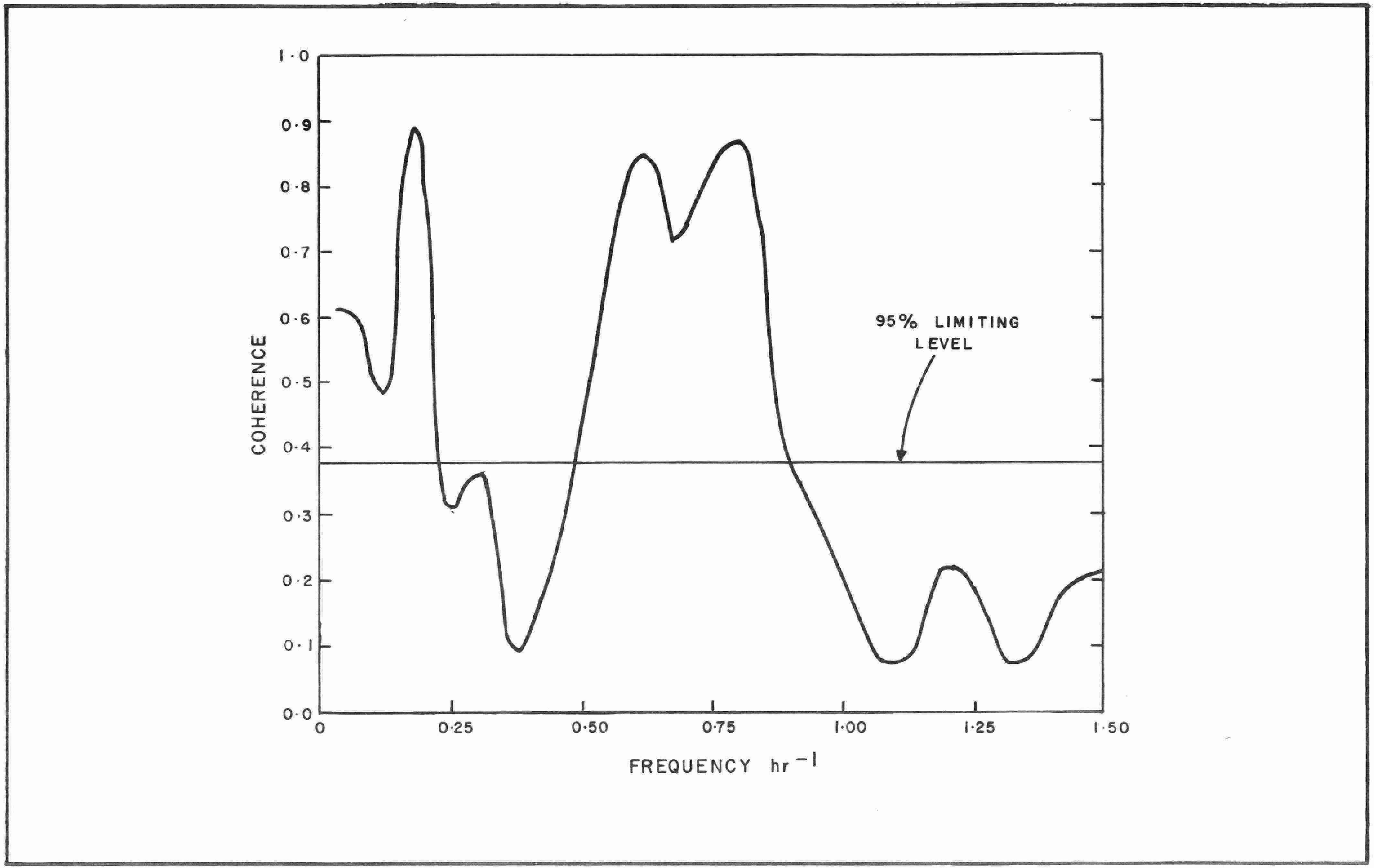


FIGURE 15: CROSS CORRELATION OF MODEL V VELOCITY AT LOCATION D WITH COMPONENT OF CURRENT METER DATA PARALLEL TO WEST GAP, LOCATION 129.

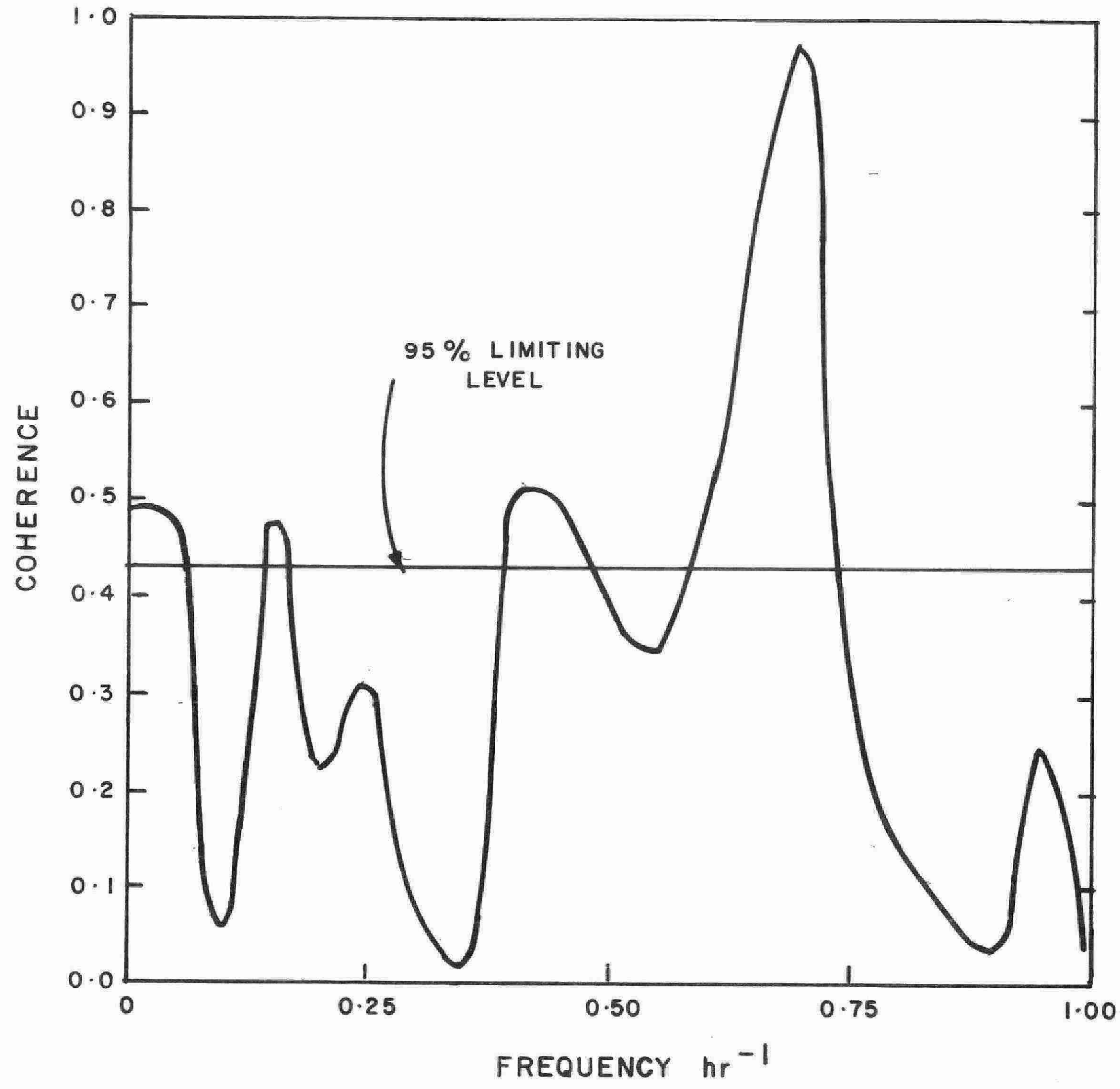


FIGURE 16: CROSS CORRELATION OF MODEL DISSOLVED SOLIDS AT LOCATION G WITH MEASURED CONDUCTIVITY, LOCATION 128.

APPENDIX

TWO-DIMENSIONAL NUMERICAL MODEL COMPUTER PROGRAM

The computer program as used in this study is capable of solving the coupled equations of momentum, continuity, and mass balance as described by Leendertse and Gritton (1971). Although so far unused, the program is capable of modelling up to five components, and provision is made for first-order decay or interaction rate constants for these components. The user is capable of selecting previously created real-time input files to describe flows and concentrations at sources and open boundaries, or to enter averaged data at model run time for these parameters. An update feature permits restarting of the program from any point in modelled time of a previous run.

The purpose of this Appendix is to describe the required input data files and provide a sample of model output, for the assistance of new users unfamiliar with the program. Descriptions of two auxilliary programs used to produce real-time input files for the model are also included. Copies of the programs (listings, cards and/or magnetic tape) are available on request from Dr. D. Poulton, Water Resources Branch.

The program is written in FORTRAN IV compatible with IBM 360 and 370 installations, and is composed of the following routines:

MAIN: establishes common blocks required for data storage for any geographical location.

HARBOR: reads in all physical data, controls calling sequence to all other subroutines and program output.

INPUT: enters real-time data (winds, source and boundary flows and concentrations) to the appropriate files.

FRITE: produces output maps of selected variables (velocities, water levels, and concentrations) at specified intervals.

HDB: calculates double-bar water depth means.

CHEZY: calculates Chezy coefficients.

UVEL: calculates U velocities and water levels at half-time steps.

VVEL: calculates V velocities and water levels at full-time steps.

PCON: calculates constituent concentrations at half-time steps.

QCON: calculates constituent concentrations at full-time steps.

Tape or disk files used by the model are given in Table A-1. File 10 is model output, whereas the other files are input. File 20 is presently created internally, while files 25 to 46 may be either created internally or copied from external tapes before model operations. All files are unformatted; files 10 and 99 normally reside on magnetic tape and the other files on disks, though this may be altered through the Job Control Language.

A complete description of the input data cards required for running the model program is given in Table A-2. Note that there are 32 types of data cards; more than one of certain cards is required, and if real-time source and boundary data are specified, certain types of data cards are omitted entirely. The formats on many of the cards contain blank fields which are useful for writing identifying information on the cards. This fact may be clarified by reference to Figure A-1, which is a complete listing of the data deck required to run the numerical model as used in this report starting at 24 hours and ending at 72 hours. Note that NSECWL, NSECF, and NSECC are 0 and consequently cards type 20, 21, 22, 24, and 26

do not appear. Card 29 must be present even though its value is not used. Although not read by the model, columns 10 to 16 of the wind cards contain the year, month, and day of wind data followed by "1" or "2" for first or second half of day.

Two additional programs have been written in this study to transform current and water quality meter recording data to the proper format accepted by the model, one for open boundaries and one for sources and sinks.

Input data used with the open boundary program are given in Table A-3. The first two data cards define the modelled period and allow for separate starting times for the current and water quality meter data records, in case there is an error in the time base for either set of recorded data (detected by a lag time to maximum cross-correlation; see report on Toronto Harbour Water Chemistry Meters). The number of spinup steps is that used in the model itself (Table A-2, Card 3) divided by the interval between successive records for real time data (NSTEPS, Table A-2, Card 6). The model time length, MINT used here is the hydrodynamic time step length multiplied by this same factor NSTEPS; this factor is used to reduce computer time for data input and is normally set so that MINT is the highest common factor in MINC and MINQ. The program decides the sign of the current meter speeds according to the compass value compared to CHANEL; if the compass reading is ambiguous (more than 45^{o} from CHANEL or CHANEL+180), the total dissolved solids (conductivity x 0.65 from the chemistry meter) is used to determine the sign. Assuming that harbour water has a higher total dissolved solids (TDS) then lake water, values of TDS above a maximum value TDSMAX produce a negative (out of harbour) current and vice versa. The total dissolved solids data are also checked against the currents; if the TDS is above TDSMAX for a positive current, TDS is set to 0.9*TDSMAX. The speeds and dissolved solids are interpolated to the model data file time step length if necessary, and output to tape files

compatible with model files 35, 36, 45 and 46. In addition, printed output is produced, one line for each output record; a star is printed to indicate an uncertain compass value, with the sign of the speed set by TDS, or the TDS being limited by positive speed. It should be noted that the present program is limited to model variables NCHANR, NCHANC, and LMAX all = 1 and must be modified for other conditions.

The source-sink program is similar to the open boundary program, except that it allows for up to 16 sources or sinks, some of which may have only average (constant) or slowly varying (e.g. daily) flow and concentration data. Input data formats are given in Table A-4. The function of spinup steps and time step length is identical to the boundary program; the sign convention for the current meters is similar, except that there is no check according to the dissolved solids value. Current meter speeds are multiplied by the cross-section area of the channel or slip at the meter location to get a flow in $M^3 s^{-1}$, and all flows are divided by the area of the grid-size to give a flow speed (m s^{-1}) into the grid square in which the source is located. The data are interpolated to the model file time step length as needed, and output to tape files compatible with model files 25 and 30. At present this program is also limited to LMAX = 1.

The regular version of these programs assumes that currents and water quality are on tape files of standard format as used in the Lake Systems Unit. For currents, this means formatted blocks of 240 words, each written in format 240F10.2, containing lines of five words representing date, time, temperature, compass (from) and speed (cm s^{-1}). For chemistry, this means unformatted blocks of 324 words each containing lines of nine words with date and time being the first two words and conductivity the last word. The other words of each line are unused.

For Toronto Harbour only, the source program was modified to replace current meter input with digitized Don River flows supplied by the Water Survey of Canada on data cards with format 16X,8F8.2.

Model Output

The numerical model produces two types of printed output and one type of magnetic tape output (File 10, Table A-1). A sample of output produced at each time step by routine HARBOR is shown in Figure A-2. Output consists of the time step number, water volume above datum in m^3, wind speed ($m\ s^{-1}$) and direction (with respect to model y-axis), row and column channel currents in $m\ s^{-1}$, first and last source flows and concentrations in $m^3 s^{-1}$ and $kg\ m^{-3}$ respectively, and output velocities ($m\ s^{-1}$), water levels above datum (m) and concentrations ($kg\ m^{-3}$) for a specified point in the model grid. The input data are presented as a verification of proper input data manipulation; this is useful in tracing data errors.

Output produced by routine FRITE is given in Figure A-3. This is produced at specified intervals (NFRITE, Table A-2, Card 5) and consists of velocities (V and U components) in $mm\ s^{-1}$, water levels above datum (mm) and concentrations ($mg\ l^{-1}$) across the entire grid line by line. For a grid more than 20 columns wide, more than one print line is produced per grid row. Lines containing no non-zero values are deleted.

Tape output on file 10 consists of groups of five records of data representing U and V velocities, water levels above datum calculated at half and full time steps and concentrations calculated at full time steps. This output is written at selected intervals (NTAPE; Table A-2, Card 6) and is used as input to cross correlation and plotting routines.

TABLE A1: Files Used by Two-dimensional Numerical Model Program

File* Name	Number of Words of Data per Record	Logical* Record Length	Block* Size	Description
FT10F001	500#	2004#	6016#	Output of velocities, water levels and concentrations at selected time steps
FT20F001	2	12	1204	Wind speed and direction
FT25F001	16	68	1704	Source flows (divided by Δx^2 where Δx is the size of grid square)
FT30F001	80	324	3244	Source concentrations
FT35F001	1	8	804	Water speed (signed scalar) at open boundary on left-hand side of model grid (row boundary)
FT36F001	1##	8##	804##	Dissolved-solids concentration corresponding to above speed
FT45F001	1	8	804	Water speed (signed scalar) at open boundary on top of model grid (column boundary)
FT46F001	1##	8##	804##	Dissolved-solids concentration corresponding to above speed
FT99F001	500#	2004#	6016#	Input of velocities, water levels and concentrations from previous model run

Notes: *Peculiar to IBM 360-370 installations. Number within file name is valid for any installation, and is used in text to refer to that file.

#Peculiar to Toronto Harbour and must be changed according to grid size of area being modelled.

##For one component in concentration model; must be increased for additional components being modelled.

TABLE A2: Numerical Model Input Card Data

Card #	Card Columns	Format	Variable Name	Description
1	9-10	I2	NR	Number of rows in model grid
	19-20	I2	NC	Number of columns in model grid
	29-30	I2	IMAX	Number of components being modelled
2	37-42	I6	IUP	= 0 when model started from rest = 1 for update run with one set of input data on file 99 >1 for update run with INITIL/NTAPE sets of input data on file 99 (see below)
	67-70	I4	INITIL	Number of time steps modelled in previous run (not used if IUP=0)
3	37-42	I6	NØSC	Number of time steps for "spin-up" of model to initial condition (winds are held constant while source and boundary flows are interpolated from zero to first value in this period)
4	37-42	I6	NSTEP	Total number of modelled steps, including "spin-up" and previously modelled steps

TABLE A2: Continued

Card #	Card Columns	Format	Variable Name	Description
5	37-42	I6	NSTEPC	Number of hydrodynamic model time steps for each mass balance time step.
	67-70	I4	NFRITE	Interval (in time steps) at which output from subroutine FRITE is desired.
6	37-42	I6	NTAPE	Interval (in time steps) at which model velocities, water levels and concentrations are written to output tape (file 10).
	67-70	I4	NSTEPS	Interval (in time steps) between successive records of real-time input data on disk (files 20-46).
7	37-42	I6	DELT	Time step length (in seconds) for hydrodynamic model.
	67-70	I4	DELX	Grid space step length (in meters)
8	31-70	4F10.0	ANINGN	Four values of Manning's N (bottom roughness) (Punch decimal point for this and all F-format variables).
9	31-38	F8.2	GRAV	Acceleration due to gravity (9.815 m s^{-2})
	56-65	F10.8	FCØR	Coriolis force (0.00010655 s^{-1} at latitude of Lake Ontario).
10	21-30	F10.8	THETA	Wind stress coefficient (dimensionless).
	46-55	F10.8	RHØA	Density of air (1.201 kg m^{-3}).
	63-70	F8.2	RHØ	Density of water (999.579 kg m^{-3}).

TABLE A2: Continued

Card #	Card Columns	Format	Variable Name	Description
11	36-45	F10.5	DISPW	Dispersion coefficient due to wind (m^2s^{-1}).
12	31-35	I5	NSEC	Number of seconds between wind readings.
	65-70	I5	NREW	Number of wind readings (speed and direction) per wind card.
13	51-52	I2	NSØR	Number of sources and sinks (present maximum 16).
14	1-64	I6I4	(RØW(I), CØL(I), I=1, NSØR)	Row and column number of each source and sink (continue on another card if needed).
15	46-61	F6.0	WDR	Wind direction constant (is angle from true north to model Y axis measured counterclockwise).
16	1-2	I2	LIMIT	Punch "15" (identifies wind card).
	17-64	12(2F2.0)	(WINDA(I),WINDB(I), I=1, NREW)	Wind direction and speed (mi/hr) Wind direction according to following 2-digit codes: (from) 11 - N, 33 - E, 55 - S, 77 - W 12 - NNE, 34 - ESE, 56 - SSW, 78 - WNW 22 - NE, 44 - SE, 66 - SW, 88 - N 23 - ENE, 45 - SSE, 67 - WSW, 89 - NNW Repeat this card for every 12 hr of wind data required, and follow with a card which has col. 1-2 blank.
17	1-3	I3	NCHANR	Number of open boundary points on left-hand edge of model grid.
	4-63	20I3	LCHANR	Row number of open boundary points (NCHANR of these).

TABLE A2: Continued

Card #	Card Columns	Format	Variable Name	Description
18	1-3	I3	NCHANC	Number of open boundary points at top edge of model grid.
	4-63	20I3	LCHANC	Column number of open boundary points (NCHANC of these).
19	1-5	I5	NSECWL	Number of seconds between card inputs of left-hand edge open boundary currents. If these data are specified on an input tape, set NSECWL = 0 and omit cards 20-22.
20	1-10	F10.0	DLIM	Any nonzero number (identifies current data).
	21-80	6F10.0	WL1	Each card has NCHANR values of left-hand edge open boundary currents for one point in time. Repeat every NSECWL seconds as required, and end with a blank card.
21	1-5	I5	NSECWL	Number of seconds between card inputs of top edge open boundary currents. If these data are specified on an input tape, set NSECWL = 0 and omit card 22.
22	1-10 21-80	F10.0 6F10.0	DLIM WL1	Identical to card #20, open boundary current data for top edge.
23	56-60	I5	NSECF	Number of seconds between card inputs of source flow data. If these data are specified in an input tape, set NSECF = 0 and omit cards 24 and 25.
24	1	A1	ALIM	"F" to identify flows.
	2-70	10(F6.0,1X)	FLØW1(I), I=0, NSØR)	flow $(m^3 s^{-1})$ for each source, two cards if NSØR >10. These cards are repeated every NSECF seconds as required and terminated with a blank card.
25	56-60	IS	NSECC	Number of seconds between card inputs of source concentrations data. If these data are specified in an input tape, set NSECC = 0 and omit cards 26.

TABLE A2: Continued

Card #	Card Columns	Format	Variable Name	Description
26	1	A1	ALIM	"C" to identify concentrations.
	2-70	10(F6.0,1X)	(CØNC1(I), I=1, NSØR)	Concentrations (mg/l) for each source, two cards if NSØR>10. These cards are repeated every NSECF seconds as required and terminated with a blank card. There must be the same number of cards 24 and 25.
27	1-75	75I1	KZ	Land-water description, NR of these cards, each with NC columns used. 4 if land point. Blank, 1, 2, 3 for water point, increasing number for increasing value of Manning's N at that grid point (according to the four N values specified on card #8).
28	1-50	5F10.2	AMB	Ambient (initial) concentration (mg/l) for each of LMAX modelled components.
29	1-50	5F10.2	LCØN	Lake (outside open boundary) concentrations (mg/l) for each of LMAX modelled components. Not used if NSECWL=0 but card must be present.
30	1-79	20(F3.0,1X)	HZ	Water depths in feet, NR of these cards, each with NC depth values across one row. If NC>20, each row requires 2 or more of these cards, according to value of NC, and there are NR such sets of cards.
31	1-50	5F10.5	ZREAC	Reaction matrix for up to LMAX components. Zero values for conservative substances.
32	1-6	2I3	IDØN,JDØN	Row and column number of a point for which velocity, water level and concentration data will be printed at each time step.

TABLE A3: Input Data to Real-Time Open Boundary Program

Card #	Card Columns	Format	Variable Name	Description
1	1-2	I2	MØNS	Start month of modelling period (01-Jan, 02-Feb, etc.)
	3-4	I2	IDAYS	Start day of modelling period.
	11-20	F10.0	TIMC	Start time for current meter data input to model.
	21-30	F10.0	TIMQ	Start time for water quality data input to model.
2	1-2	I2	MØNE	End month of modelling period.
	3-4	I2	IDAYE	End day of modelling period.
	5-6	I2	IYR	Year of modelling period.
3	1-3	I3	NØSC	Number of "spin-up" steps.
4	1-2	I1	MINC	Current meter recording interval (min.)
	3-4	I2	MINQ	Chemistry meter recording interval (min.)
	5-6	I2	MINT	Model time length between successive inputs of real time data (min.)
5	1-10	F10.0	CHANEL	Compass orientation from which positve (into modelled area) currents flow.
	11-20	F10.0	TDSMAX	Maximum dissolved solids (conductivity *0.65) concentration for positive current. If current is positive and TDS > TDSMAX, TDS is set to 0.9 * TDSMAX.

TABLE A4: Input Data to Real-Time Source-Sink Program

Card #	Card Columns	Format	Variable Name	Description
1,2,3				Identical to cards described in Table A3.
4	1-10	F10.0	DELX	Grid space step length (in metres).
5	1-2	I2	NMETER	Number of metered sources (current and chemistry meter data available).
	3-4	I2	NCØNST	Number of constant sources (only average data available).
	5-6	I2	NVAR	Number of slowly varying sources (have data read in from cards at a specified interval MHRV)
6	1-2	I2	MINC	Current meter recording interval (min.).
	3-4	I2	MINQ	Chemistry meter recording interval (min.). This card is repeated for each metered source.
7	1-2	I2	MHRV	Number of hours between readings of slowing varying sources.
	3-4	I2	MINT	Model time length between successive inputs of real time data (min.).
8	1-10	F10.0	CSECT	Cross-section area of water channel, slip or outfall at current meter location.
	11-20	F10.0	CHANEL	Compass orientation from which positive (into modelled area) currents flow. This card is repeated for each metered source.

TABLE A4: continued

Card #	Card Columns	Format	Variable Name	Description
9	1-72	12F6.0	FLØW	Flow speeds (m^3/s) of constant sources (Negative for sinks).
10	1-72	12F6.0	CØN	Concentrations (mg/l) of constant sources.
11	1-72	12F6.0	FLØV	Flow speeds (m^3/s) of slowly varying sources (negative for sinks).
12	1-72	12F6.0	CØNV	Concentrations (mg/l) of slowly varying sources. One reading on each of cards 11 and 12 every MHRV hours, go to entra cards if necessary. Repeat cards 11 and 12 for each slowly varying source. If no slowly varying source is present, omit these cards.

FIGURE A1

NUMERICAL MODEL INPUT DATA LISTING

```
# ROWS =25 # COLS=20 LMAX = 01
UPDATE RUN STARTING AT (4+24) HR              1                              5040
NUMBER OF STEPS TO INITIAL CONDITION        720
TOTAL NUMBER OF STEPS - 72 HRS            13680     (INCLUDES NOSC)
NUMBER OF STEPS PER CONCN STEP                3     BETWEEN FRITE STEPS     180
NUMBER OF STEPS BETWEEN TAPE RECORDS          6     BETWEEN DISK RECORDS     30
TIME STEP LENGTH IN SECONDS     20.0       SPACE STEP IN METERS       152.4
FOUR VALUES OF MANNINGS N ARE 0.040       0.045       0.050       0.060
ACCELERATION DUE TO GRAVITY     9.815    CORIOLIS FORCE = .00010655
WIND STRESS COEFF = 0.00320    DENSITY OF AIR= 1.20142  DENSITY OF WATER=999.579
DISPERSION COEFF DUE TO WIND =        0.929
SECONDS BETWEEN WIND READINGS= 3600              ONE CARD CONTAINS        12 R
NUMBER OF MUNICIPAL & INDUSTRIAL SOURCES & SINKS =02 ROW & COL # OF EACH
  24   12   24   07
WIND DIRECTION CONSTANT FOR TORONTO HARBOR IS 235.0  DEGREES.
15         751108266157716771977197817781778157706670467046704880 5
15         751109188031203230533052304230422082209331533143309331 0
15         751109234133312331433112312331434142314231823182320231 7
15         751110123162316231433122314231433133413551745164517551 8
15         751110266236734673166356734673267336734673167256726673 1
  /* END OF WIND
  1 21
  1 18
      NSECWL=0 FOR TAPE INPUT OF OPEN BOUNDARIES
      NSECF = 0 FOR TAPE INPUT OF SOURCES & SINKS
      NSECC = 0 FOR TAPE INPUT OF SOURCE CONCENTRATIONS
4444444444444444 44
4444444444444444 44
4444444444224421 44
4444444442222421 1 14
44444442222222 1  44
44444222111111 1 444
444442211111111  444
4444221111111 11 4444
444422111111 1   4444
44422111111 11  44444
443221111111 1  44444
443221111111   444444
44432111111    444444
444322111 1    444444
44432211 1     444444
44432111 1     444444
4443221 1      444444
4444221        444444
444421        4444444
444421        4444444
              4444444
4444          4444444
444444 44441144444444
444444 4444144444444
4444444444444444444444
221.1                                                            AMB = 340 UMHO
195.                                                             LCON = 300 UMHO
                                                                                                    22    22
                                                                 8    10     5          13    16    22    22
                                                     5     5    12    28    18     7    17    20    27    14    12
                                         7    13    13    16    31    25    27    16    26    24    27    14     9
                             5    16    14    24    24    29    31    27    30    26    27    27    27    14
                             5    14    25    26    27    25    33    26    28    32    27    27    26
                       5    13    28    30    29    28    27    33    36    29    29    27    27    20
                       7    13    25    28    31    31    31    38    38    30    24    27    21
                 5    10    17    25    28    28    28    27    30    25    25    27    27    22
           6     8    14    22    26    27    28    27    26    24    22    27    27    20
           5     9    28    24    26    28    27    26    26    24    27    27    27    20
           5     7    22    23    26    27    25    27    26    26    27    27    27
                 5    15    24    25    26    27    25    24    27    27    27    27
                 5    11    21    23    25    25    21    27    27    27    27    27
                 5     9    21    21    23    23    27    27    27    27    27    27
                 5     9    23    20    22    27    27    27    27    27    27    27
                 5     7    19    20    26    27    27    27    27    27    27    27
                       5    16    19    27    27    27    27    27    27    27    27
                       5    18    27    27    27    27    27    27    27    27
    29    29    29    29    29    27    27    27    27    27    27    27    27
    29    29    29    29    29    27    27    27    27    27    27    27    27
                      13    13    27    27    27    27    27    27    27    22
                                  20    20                      26    22    22
                                  20    20                      26    18

                                                                 ALL REACTION TERMS ARE ZERO
 18 06
//
```

FIGURE A2

NUMERICAL MODEL OUTPUT BY SUBROUTINE HARBOR

TIME STEP	TOTAL VOLUME	WIND SPEED	WIND DIR	CHANNEL FLOW 1 ROW	CHANNEL FLOW 1 COL	SOURCE 1 FLOW	SOURCE 1 CONC	SOURCE NSOR FLOW	SOURCE NSOR CONC	U 18, 6	V 18, 6	WATER LEVEL	CONC 18, 6
13441	0.2427D 07	7.92	62.4	0.009	0.048	32.30	0.310	-50.00	0.0	0.0051	-0.0062	0.46225	0.23183
13442	0.2428D 07	7.92	62.3	0.009	0.047	32.32	0.310	-50.00	0.0	0.0057	-0.0049	0.46315	0.23183
13443	0.2429D 07	7.92	62.1	0.009	0.047	32.34	0.310	-50.00	0.0	0.0065	-0.0034	0.46428	0.23190
13444	0.2430D 07	7.93	62.0	0.009	0.046	32.36	0.310	-50.00	0.0	0.0071	-0.0024	0.46555	0.23190
13445	0.2431D 07	7.93	61.9	0.009	0.046	32.38	0.310	-50.00	0.0	0.0075	-0.0021	0.46733	0.23190
13446	0.2432D 07	7.93	61.8	0.009	0.046	32.40	0.309	-50.00	0.0	0.0077	-0.0020	0.46971	0.23204
13447	0.2432D 07	7.93	61.6	0.009	0.045	32.42	0.309	-50.00	0.0	0.0076	-0.0018	0.47205	0.23204
13448	0.2433D 07	7.94	61.5	0.009	0.045	32.44	0.309	-50.00	0.0	0.0072	-0.0021	0.47390	0.23204
13449	0.2434D 07	7.94	61.4	0.009	0.045	32.45	0.309	-50.00	0.0	0.0069	-0.0032	0.47551	0.23217
13450	0.2435D 07	7.94	61.3	0.009	0.044	32.47	0.308	-50.00	0.0	0.0070	-0.0041	0.47679	0.23217
13451	0.2436D 07	7.94	61.1	0.009	0.044	32.49	0.308	-50.00	0.0	0.0075	-0.0048	0.47741	0.23217
13452	0.2437D 07	7.95	61.0	0.009	0.043	32.51	0.308	-50.00	0.0	0.0083	-0.0056	0.47788	0.23222
13453	0.2438D 07	7.95	60.9	0.009	0.043	32.53	0.308	-50.00	0.0	0.0093	-0.0066	0.47869	0.23222
13454	0.2439D 07	7.95	60.8	0.010	0.043	32.55	0.307	-50.00	0.0	0.0103	-0.0074	0.47942	0.23222
13455	0.2439D 07	7.95	60.6	0.010	0.042	32.57	0.307	-50.00	0.0	0.0109	-0.0084	0.48008	0.23227
13456	0.2440D 07	7.96	60.5	0.010	0.042	32.59	0.307	-50.00	0.0	0.0108	-0.0096	0.48125	0.23227
13457	0.2441D 07	7.96	60.4	0.010	0.042	32.61	0.307	-50.00	0.0	0.0103	-0.0105	0.48241	0.23227
13458	0.2442D 07	7.96	60.3	0.010	0.041	32.62	0.306	-50.00	0.0	0.0095	-0.0114	0.48258	0.23231
13459	0.2443D 07	7.96	60.1	0.010	0.041	32.64	0.306	-50.00	0.0	0.0085	-0.0127	0.48189	0.23231
13460	0.2444D 07	7.97	60.0	0.010	0.040	32.66	0.306	-50.00	0.0	0.0076	-0.0145	0.48087	0.23231
13461	0.2444D 07	7.97	59.9	0.010	0.040	32.68	0.306	-50.00	0.0	0.0073	-0.0162	0.47942	0.23222
13462	0.2445D 07	7.97	59.8	0.010	0.040	32.70	0.305	-50.00	0.0	0.0072	-0.0181	0.47756	0.23222
13463	0.2446D 07	7.97	59.6	0.010	0.039	32.72	0.305	-50.00	0.0	0.0075	-0.0205	0.47610	0.23222
13464	0.2447D 07	7.98	59.5	0.010	0.039	32.74	0.305	-50.00	0.0	0.0081	-0.0228	0.47559	0.23215
13465	0.2448D 07	7.98	59.4	0.010	0.038	32.76	0.305	-50.00	0.0	0.0092	-0.0241	0.47564	0.23215
13466	0.2448D 07	7.98	59.3	0.010	0.038	32.78	0.305	-50.00	0.0	0.0104	-0.0248	0.47563	0.23215
13467	0.2449D 07	7.98	59.1	0.010	0.038	32.79	0.304	-50.00	0.0	0.0113	-0.0254	0.47554	0.23214
13468	0.2450D 07	7.99	59.0	0.010	0.037	32.81	0.304	-50.00	0.0	0.0118	-0.0250	0.47530	0.23214
13469	0.2451D 07	7.99	58.9	0.010	0.037	32.83	0.304	-50.00	0.0	0.0120	-0.0231	0.47414	0.23214
13470	0.2452D 07	7.99	58.8	0.010	0.037	32.85	0.304	-50.00	0.0	0.0118	-0.0210	0.47196	0.23202
13471	0.2452D 07	7.99	58.6	0.010	0.035	32.87	0.303	-50.00	0.0	0.0113	-0.0193	0.46996	0.23202
13472	0.2453D 07	8.00	58.5	0.010	0.033	32.89	0.303	-50.00	0.0	0.0107	-0.0168	0.46866	0.23202
13473	0.2454D 07	8.00	58.4	0.010	0.031	32.91	0.303	-50.00	0.0	0.0106	-0.0136	0.46745	0.23192
13474	0.2454D 07	8.00	58.3	0.010	0.029	32.93	0.303	-50.00	0.0	0.0109	-0.0109	0.46640	0.23192
13475	0.2455D 07	8.00	58.1	0.010	0.027	32.95	0.302	-50.00	0.0	0.0114	-0.0089	0.46620	0.23192
13476	0.2456D 07	8.01	58.0	0.010	0.025	32.96	0.302	-50.00	0.0	0.0119	-0.0068	0.46672	0.23192
13477	0.2456D 07	8.01	57.9	0.010	0.023	32.98	0.302	-50.00	0.0	0.0118	-0.0051	0.46761	0.23192
13478	0.2457D 07	8.01	57.8	0.010	0.021	33.00	0.302	-50.00	0.0	0.0110	-0.0041	0.46896	0.23192
13479	0.2457D 07	8.01	57.6	0.010	0.019	33.02	0.301	-50.00	0.0	0.0097	-0.0034	0.47056	0.23201
13480	0.2457D 07	8.02	57.5	0.011	0.017	33.04	0.301	-50.00	0.0	0.0082	-0.0028	0.47204	0.23201
13481	0.2458D 07	8.02	57.4	0.011	0.016	33.06	0.301	-50.00	0.0	0.0067	-0.0028	0.47350	0.23201
13482	0.2458D 07	8.02	57.3	0.011	0.014	33.08	0.301	-50.00	0.0	0.0058	-0.0030	0.47481	0.23211
13483	0.2458D 07	8.02	57.1	0.011	0.012	33.10	0.300	-50.00	0.0	0.0058	-0.0030	0.47568	0.23211
13484	0.2459D 07	8.03	57.0	0.011	0.010	33.12	0.300	-50.00	0.0	0.0064	-0.0030	0.47645	0.23211
13485	0.2459D C7	8.03	56.9	0.011	0.008	33.13	0.300	-50.00	0.0	0.0073	-0.0033	0.47765	0.23219
13486	0.2459D 07	8.03	56.8	0.011	0.006	33.15	0.300	-50.00	0.0	0.0089	-0.0040	0.47912	0.23219
13487	0.2459D 07	8.03	56.6	0.011	0.004	33.17	0.299	-50.00	0.0	0.0108	-0.0046	0.48054	0.23219
13488	0.2459D 07	8.04	56.5	0.011	0.002	33.19	0.299	-50.00	0.0	0.0121	-0.0054	0.48218	0.23230
13489	0.2459D 07	8.04	56.4	0.011	0.000	33.21	0.299	-50.00	0.0	0.0126	-0.0063	0.48415	0.23230
13490	0.2459D 07	8.04	56.3	0.011	-0.002	33.23	0.299	-50.00	0.0	0.0128	-0.0072	0.48574	0.23230
13491	0.2459D 07	8.04	56.1	0.011	-0.004	33.25	0.299	-50.00	0.0	0.0125	-0.0084	0.48640	0.23237
13492	0.2459D 07	8.05	56.0	0.011	-0.005	33.27	0.298	-50.00	0.0	0.0115	-0.0102	0.48663	0.23237
13493	0.2459D 07	8.05	55.9	0.011	-0.007	33.29	0.298	-50.00	0.0	0.0107	-0.0118	0.48671	0.23237
13494	0.2459D 07	8.05	55.8	0.011	-0.009	33.30	0.298	-50.00	0.0	0.0108	-0.0124	0.48595	0.23233
13495	0.2459D 07	8.05	55.6	0.011	-0.011	33.32	0.298	-50.00	0.0	0.0113	-0.0126	0.48417	0.23233
13496	0.2458D 07	8.06	55.5	0.011	-0.013	33.34	0.297	-50.00	0.0	0.0117	-0.0136	0.48235	0.23233
13497	0.2458D C7	8.06	55.4	0.011	-0.015	33.36	0.297	-50.00	0.0	0.0121	-0.0149	0.48116	0.23223
13498	0.2458D 07	8.06	55.3	0.011	-0.017	33.38	0.297	-50.00	0.0	0.0123	-0.0158	0.48046	0.23223
13499	0.2457D 07	8.06	55.1	0.011	-0.019	33.40	0.297	-50.00	0.0	0.0119	-0.0169	0.48020	0.23223
13500	0.2457D 07	8.07	55.0	0.011	-0.021	33.42	0.296	-50.00	0.0	0.0112	-0.0185	0.48045	0.23222

FIGURE A3

NUMERICAL MODEL OUTPUT BY SUBROUTINE FRITE

*** V

1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-13	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	-19	18	0	0	-34	0	12	0	0
4	1	0	0	0	0	0	0	0	0	0	-44	-30	-10	66	0	-51	-45	0	59	0	0
5	1	0	0	0	0	0	0	0	-31	-41	-40	-11	4	46	-2	6	-28	19	0	0	0
6	1	0	0	0	0	0	-19	-55	-23	-41	-23	-13	3	49	-3	-1	-14	24	0	0	0
7	1	0	0	0	0	0	-21	-19	-13	-51	-7	-21	-25	44	12	0	-9	0	0	0	0
8	1	0	0	0	0	-19	-10	-42	-12	-46	-24	3	1	30	4	-18	-1	0	0	0	0
9	1	0	0	0	0	-37	-3	-22	3	-43	-24	4	0	2	-17	-4	0	0	0	0	0
10	1	0	0	0	-50	-26	-10	-30	15	-6	-21	10	-16	0	-15	-7	0	0	0	0	0
11	1	0	0	42	-5	-18	-6	-30	0	-17	-7	-6	-12	4	-26	0	0	0	0	0	0
12	1	0	0	0	25	-41	22	-18	-25	-12	3	-11	-5	-5	-27	0	0	0	0	0	0
13	1	0	0	0	65	-12	-4	-25	-18	-6	-4	-12	-6	-11	-20	0	0	0	0	0	0
14	1	0	0	0	79	-9	-8	-19	-15	-13	-5	-3	-5	-12	-21	0	0	0	0	0	0
15	1	0	0	0	73	18	-23	-36	-3	-16	-7	-6	-5	-10	-4	0	0	0	0	0	0
16	1	0	0	0	77	11	-35	-45	0	-3	-11	-5	0	-5	-5	0	0	0	0	0	0
17	1	0	0	0	0	52	-30	-29	-6	-13	-10	-10	-1	-9	9	0	0	0	0	0	0
18	1	0	0	0	0	19	-18	-9	-9	-20	-1	-8	0	-10	0	0	0	0	0	0	0
19	1	0	0	0	0	-20	2	-3	-2	-21	9	-18	-2	-15	0	0	0	0	0	0	0
20	1	0	0	0	0	-18	21	-1	2	-26	6	-10	-5	-20	0	0	0	0	0	0	0
21	1	0	0	0	0	19	38	-36	8	-25	9	-6	-15	-21	0	0	0	0	0	0	0
22	1	0	0	0	0	0	0	29	0	0	0	0	-15	-26	0	0	0	0	0	0	0
23	1	0	0	0	0	0	0	43	0	0	0	0	-34	0	0	0	0	0	0	0	0

$$$ U

3	1	0	0	0	0	0	0	0	0	0	0	0	21	0	0	0	26	14	0	0	0
4	1	0	0	0	0	0	0	0	0	0	59	63	48	0	0	44	58	43	10	0	0
5	1	0	0	0	0	0	0	0	29	80	84	50	42	52	70	4	-16	-41	0	0	0
6	1	0	0	0	0	0	5	54	40	42	22	19	24	21	20	26	8	0	0	0	0
7	1	0	0	0	0	0	7	-16	-25	-13	-30	-16	14	12	-5	-10	-19	0	0	0	0
8	1	0	0	0	0	10	-6	12	9	8	28	1	-23	-15	-7	9	0	0	0	0	0
9	1	0	0	0	0	16	3	-17	-33	-41	-44	-37	-37	-6	16	2	0	0	0	0	0
10	1	0	0	0	44	25	26	31	20	-15	-19	-23	-7	-3	-3	0	0	0	0	0	0
11	1	0	0	-43	-44	-41	-40	-37	-21	-11	-24	-6	-6	-8	4	0	0	0	0	0	0
12	1	0	0	30	-8	10	-17	-27	-2	-7	-17	-11	-15	-3	0	0	0	0	0	0	0
13	1	0	0	0	-16	-41	-11	-4	-10	-14	-5	-3	-1	5	0	0	0	0	0	0	0
14	1	0	0	0	4	0	4	-1	-3	2	4	-2	-2	0	0	0	0	0	0	0	0
15	1	0	0	0	18	-8	6	21	8	13	15	18	18	16	0	0	0	0	0	0	0
16	1	0	0	0	3	7	21	27	21	7	11	9	3	-1	0	0	0	0	0	0	0
17	1	0	0	0	77	8	1	-11	-3	6	5	10	11	15	0	0	0	0	0	0	0
18	1	0	0	0	0	28	11	-9	-5	1	-8	-10	-11	-9	0	0	0	0	0	0	0
19	1	0	0	0	0	25	1	-2	-9	-8	-18	-7	-5	0	0	0	0	0	0	0	0
20	1	0	0	0	0	11	-9	-11	-15	-9	-5	-12	-6	0	0	0	0	0	0	0	0
21	1	11	11	11	11	-26	-45	-8	-13	-13	-14	-17	-4	0	0	0	0	0	0	0	0
22	1	0	0	0	0	27	62	-2	7	-16	-5	-9	-6	0	0	0	0	0	0	0	0
23	1	0	0	0	0	0	0	0	0	0	0	0	22	0	0	0	0	0	0	0	0

*** ZETA

1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	442	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	448	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	468	468	0	0	467	465	459	0	0
4	1	0	0	0	0	0	0	0	0	0	469	469	469	470	0	471	470	468	466	468	0
5	1	0	0	0	0	0	0	0	470	470	470	470	470	471	472	472	472	471	468	0	0
6	1	0	0	0	0	0	471	471	471	471	471	471	471	472	473	473	474	473	0	0	0
7	1	0	0	0	0	0	472	472	471	471	471	471	472	472	473	474	474	474	0	0	0
8	1	0	0	0	0	471	472	472	471	471	472	472	472	473	474	474	475	0	0	0	0
9	1	0	0	0	0	471	472	472	472	472	472	472	472	473	474	475	475	0	0	0	0
10	1	0	0	0	473	472	472	472	472	472	472	472	473	473	474	475	0	0	0	0	0
11	1	0	0	473	473	473	472	472	472	473	473	473	474	474	474	475	0	0	0	0	0
12	1	0	0	474	473	473	473	473	473	474	474	474	475	475	475	0	0	0	0	0	0
13	1	0	0	0	473	473	473	474	474	475	475	475	476	477	477	0	0	0	0	0	0
14	1	0	0	0	472	473	474	475	476	476	477	477	478	478	479	0	0	0	0	0	0
15	1	0	0	0	473	474	475	476	477	473	479	479	480	480	481	0	0	0	0	0	0

FIGURE A3, continued

16	1	0	0	0	473	476	477	478	479	480	481	481	482	482	483	0	0	0	0	0	0
17	1	C	0	0	475	477	478	480	481	481	482	483	484	484	484	0	0	0	0	0	0
18	1	0	0	0	0	479	480	481	482	483	483	484	485	485	485	0	0	0	0	0	0
19	1	0	0	0	0	481	482	482	483	483	484	485	485	486	0	0	0	0	0	0	0
20	1	0	0	C	0	483	483	483	483	484	485	485	485	485	0	0	0	0	0	0	0
21	1	487	487	487	485	484	483	483	484	484	485	485	484	485	0	0	0	0	0	0	0
22	1	0	0	0	0	484	483	482	484	484	485	485	484	484	0	0	0	0	0	0	0
23	1	0	0	0	0	0	0	480	0	0	0	0	482	483	0	0	0	0	0	0	0
24	1	0	0	0	0	0	0	478	0	0	0	0	481	0	0	0	0	0	0	0	0

* CONQ= 1

1	1	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	210	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	209	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	227	226	0	0	213	211	209	0	0
4	1	0	0	0	0	0	0	0	0	0	227	227	227	226	0	218	214	213	211	209	0
5	1	0	0	0	0	0	0	0	227	227	227	227	227	226	227	220	210	211	211	0	0
6	1	0	0	0	0	0	228	227	227	226	226	227	227	226	225	219	214	211	0	0	0
7	1	0	0	0	0	0	227	226	226	226	227	227	227	226	222	214	209	210	0	0	0
8	1	0	0	0	0	228	227	226	226	226	226	226	226	225	221	216	213	0	0	0	0
9	1	0	0	0	0	228	227	226	226	226	226	226	226	225	222	219	215	0	0	0	0
10	1	0	0	0	228	228	227	226	226	226	226	227	225	224	221	219	0	0	0	0	0
11	1	C	0	228	227	226	225	225	226	224	225	225	223	223	220	219	0	0	0	0	0
12	1	C	0	229	229	229	227	225	224	224	224	223	222	222	221	0	0	0	0	0	0
13	1	0	0	0	229	228	227	226	224	223	223	223	222	222	222	0	0	0	0	0	0
14	1	C	0	0	229	230	230	229	225	224	223	223	222	222	222	0	0	0	0	0	0
15	1	C	0	0	229	230	230	229	227	225	224	224	223	223	223	0	0	0	0	0	0
16	1	0	0	0	230	230	230	229	228	227	225	224	224	223	223	0	0	0	0	0	0
17	1	C	0	0	230	230	230	228	227	227	226	225	224	224	223	0	0	0	0	0	0
18	1	0	0	0	0	231	232	229	228	228	226	225	224	224	223	0	0	0	0	0	0
19	1	0	0	0	0	232	232	230	230	231	226	227	225	227	0	0	0	0	0	0	0
20	1	0	0	0	0	231	232	232	232	235	228	232	231	237	0	0	0	0	0	0	0
21	1	218	215	216	219	229	234	233	235	238	234	242	251	259	0	0	0	0	0	0	0
22	1	0	0	0	0	229	232	233	236	239	240	253	294	298	0	0	0	0	0	0	0
23	1	0	0	0	0	0	0	233	0	0	0	0	381	352	0	0	0	0	0	0	0
24	1	0	0	0	0	0	0	233	0	0	0	0	375	0	0	0	0	0	0	0	0